MATH464, Sec. 0101: Transform Methods
Department of Mathematics, UMCP **Fall 2022**
Homework 3 **Posted**: Thursday, 09/15/22
Due: Thursday, 09/22/22 **IN CLASS**

Answer all questions. Make sure that you explain all your steps and justify your answers. Each problem is worth 10 points (equally distributed among its parts). **Total number of points: 80**

Note: The use of Matlab, or any other software, is strictly NOT permitted.

17. Compute the Fourier coefficients, and expand in Fourier series the 1-periodic function $f(x) = x^2$, for $0 < x < 1$.

18. Compute the Fourier coefficients, and expand in Fourier series the 1-periodic function $f(x) = x^3$, for $0 < x < 1$.

19. (i) Compute the Fourier coefficients, and expand in Fourier series the 1-periodic function

$$f(x) = \begin{cases} x & \text{if } 0 < x < \frac{1}{2} , \\ a & \text{if } \quad x = \frac{1}{2} , \\ 0 & \text{if } \frac{1}{2} < x < 1 . \end{cases}$$

(ii) What should the const. a be so that the Fourier series converges to $f(x)$ for every $0 < x < 1$?

20. (i) Compute the Fourier coefficients, and expand in Fourier series the 1- periodic function

$$f(x) = \begin{cases} 4x , & 0 < x \le 1/4 , \\ 2 - 4x , & 1/4 < x \le 1/2 , \\ 4x - 2 , & 1/2 < x \le 3/4 , \\ 4 - 4x , & 3/4 < x < 1 . \end{cases}$$

(ii) Is the Fourier series convergent for every $0 < x < 1$? Explain.

In Problems 21 and 22 the functions are extended by periodicity outside the interval of definition. Compute the Fourier coefficients, and expand in Fourier series the following 1-periodic functions.

21. $f(x) = \sin(\pi x)$, for $-1/2 < x < 1/2$.

22. $f(x) = \cos(\pi x)$, for $-1/2 < x < 1/2$.

23. Consider the 1-periodic function $f : [0, 1) \to \mathbb{R}$ whose Fourier coefficients $F[n]$ decay to 0 as $|n| \to \infty$, and are bounded above by $|F[n]| \le \frac{10}{|n|}$, for $|n| \ge 1$. Estimate the minimum degree N of the symmetric trigonometric polynomial $A_N(x) = \sum_{k=-N}^{N} F[k]\, e^{2\pi ikx}$ that approximates $f(x)$ with a 5% mean square error.

24. Consider the 1-periodic function $f : [0, 1) \to \mathbb{R}$ whose Fourier coefficients $F[n]$ decay to 0 as $|n| \to \infty$, and are bounded above by $|F[n]| \le \frac{10}{n^4}$, for $|n| \ge 1$. Estimate the minimum degree N of the symmetric trigonometric polynomial $A_N(x) = \sum_{k=-N}^{N} F[k]\, e^{2\pi ikx}$ that approximates $f(x)$ with a 1% mean square error.

Notes on Problems 23 and 24: You may use the inequality (for what a?) $\sum_{n=N+1}^{\infty} \frac{1}{n^a} \le \frac{1}{(a-1)N^{a-1}}$.
Mean square error is defined as $\int_0^1 |f(x) - A_N(x)|^2\, dx$.
1%=0.01 , 5%=0.05.