

MATH464, Sec. 0101: Transform Methods
Department of Mathematics, UMCP
Homework 4

Fall 2022

Posted: Thursday, 09/22/22

Due: Thursday, 09/29/22 **IN CLASS**

Answer all questions. Make sure that you explain all your steps and justify your answers. Each problem is worth 10 points (equally distributed among its parts). **Total number of points: 50**

Note: The use of Matlab, or any other software, is strictly NOT permitted.

25. Consider the function $f : [-1/2, 1/2) \rightarrow \mathbb{R}$ whose Fourier coefficients (in the complex-exponential Fourier series) satisfy $|F[k]| \leq C \cdot 10^{-|k|}$ for all $k \in \mathbb{Z}$. If $C = 90$, how many terms, N , are needed so that the N -partial Fourier sum $\sum_{k=-N}^N F[k]e^{2\pi i k x}$ results in a (pointwise) error not larger than 0.0002 at every point x ? **Hint:** Consider the geometric series $\sum_{k=N+1}^{\infty} \lambda^k$ for a suitable λ with $|\lambda| < 1$.

Compute the Fourier transforms of the following functions $f : \mathbb{R} \rightarrow \mathbb{R}$ (Problems 26-29):

26.

$$f(x) = \begin{cases} 1 & \text{if } 1 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

27. $f(x) = e^{-a|x|}$ for some $a > 0$.

28.

$$f(x) = \frac{1}{x^2 + a^2},$$

for some $a > 0$. **Hint:** Use synthesis formula and Problem 27.

29.

$$f(x) = \begin{cases} \sin(2\pi x) & \text{if } 1 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$