

Answer all questions. Make sure that you explain all your steps and justify your answers. Each problem is worth 10 points (equally distributed among its parts). **Total number of points: 80**

Note: The use of Matlab, or any other software, is strictly NOT permitted.

In Problems 38-40, use your knowledge of Fourier analysis to find a function f that satisfies the indicated integral equation.

38.

$$\int_0^\infty f(u) \cos(2\pi ux) du = e^{-x}, \quad 0 < x < \infty.$$

39.

$$\int_0^\infty f(u) \sin(2\pi ux) du = \begin{cases} 1, & \text{if } 0 < x < 1, \\ 0, & \text{if } 1 < x < \infty. \end{cases}$$

40.

$$\int_{-\infty}^\infty f(u) e^{iux} du = e^{-\pi x^2}, \quad 0 < x < \infty.$$

41. Consider the following differential equation:

$$-f''(x) + f(x) = e^{-2|x|}, \quad -\infty < x < \infty.$$

(i)[5pts] Find the Fourier transform $F(s)$ of the solution $f(x)$ which is absolutely integrable (i.e., $\int_{-\infty}^\infty |f(x)| dx < \infty$).

(ii)[5pts] Obtain the function f .

42. Find the Fourier transform of the function $f : \mathbb{R} \rightarrow \mathbb{R}$ where

$$f(x) = \int_{-1/2}^{1/2} e^{-2\pi(x-u)^2} du.$$

43. Let $f_0, f_1 : \mathbb{R} \rightarrow \mathbb{R}$ be the functions defined by

$$f_0(x) = e^{-x^2}, \quad f_1(x) = x e^{-x^2}.$$

Compute the following convolutions:

(i)[5pts] $f_0 * f_0$.

(ii)[5pts] $f_0 * f_1$.

44. Find a nonzero function f on $\mathbb{R}$ that satisfies

$$\int_{-\infty}^\infty f(u) f(x-u) du = f(x), \quad -\infty < x < \infty.$$

45. Find a continuous function $f : \mathbb{R} \rightarrow \mathbb{R}$ that satisfies

$$-f'(x) + f(x) = \begin{cases} e^{-x}, & \text{if } x > 0, \\ 0, & \text{if } x < 0. \end{cases}$$