

MATH464, Sec. 0101: Transform Methods
Department of Mathematics, UMCP
Homework 8

Fall 2022

Posted: Wednesday, 11/02/22
Due: Thursday, 11/10/22 **IN CLASS**

Answer all questions. Show all your steps and justify your answers. **Total number of pts: 80**

Note: The use of Matlab, or any other software, is strictly NOT permitted.

In Problems 51–53: Find the value of each of the displayed “integrals”, I . (These are symbolic integrals, not Riemann or Lebesgue integrals.)

51. (a)[5pts]

$$I = \int_{-\infty}^{\infty} \delta(x - 3) e^{-\pi x^4} dx .$$

(b)[5pts]

$$I = \int_{-\infty}^{\infty} \delta'(x - 1) e^{-\pi x^2} dx .$$

52. (a)[10pts]

$$I = \int_{-\infty}^{\infty} \delta''(x) e^{-\pi x^2} dx .$$

(b)[10pts]

$$I = \int_{-\infty}^{\infty} \delta'''(x) e^{-\pi x^2} dx .$$

53. (a)[5pts]

$$I = \int_{-\infty}^{\infty} [\cos(\pi x) \delta(x)] e^{-\pi x^2} dx .$$

(b)[5pts]

$$I = \int_{-\infty}^{\infty} [\sin(\pi x) \delta'(x)] e^{-\pi x^2} dx .$$

In Problems 54–57: Find and simplify the functional $f\{\phi\}$, where ϕ is test function of the Schwartz class ($\phi \in \mathcal{S}$), that is used to represent the generalized function f indicated below.

54. (a)[5pts] $f(x) = \delta'(2x)$. (b)[5pts] $f(x) = \Pi'(x)$.

55. (a)[5pts] $f(x) = \delta''(x - 6)$. (b)[5pts] $f(x) = \delta'''(x - 5)$.

56. [10pts] $\widehat{f}(s) = \delta''(s)$.

57. [10pts]

$$f(x) = \frac{27}{(x^2 + 5x + 4)^2} = \frac{2}{x+4} + \frac{3}{(x+4)^2} - \frac{2}{x+1} + \frac{3}{(x+1)^2} .$$