

MATH464, Sec. 0101: Transform Methods
Department of Mathematics, UMCP
Homework 9

Fall 2022

Posted: Saturday, 11/12/22

Due: Tuesday, 11/29/22 IN CLASS

Answer all questions. Show all your steps and justify your answers. **Total number of pts: 80**

Note: The use of Matlab, or any other software, is strictly NOT permitted.

58. [10pts] Compute the Fourier transforms of the following generalized functions:
- (a)[5pts] $f(x) = x \cdot \delta'(x)$;
 - (b)[5pts] $f(x) = \sin^2(\pi x)$.
59. [15pts] Let a, b, c, d be real numbers with $a > 0$ and $c > 0$. Find a simple representation for the convolution product $\beta * \gamma$ if:
- (a)[5pts] $\beta(x) = \delta(ax + b)$, $\gamma(x) = \sin(cx + d)$;
 - (b)[5pts] $\beta(x) = e^{-ax^2}$, $\gamma(x) = \sin(cx + d)$;
 - (c)[5pts] $\beta(x) = \delta'(ax + b)$, $\gamma(x) = \operatorname{sgn}(x)$;
60. [20pts] Find all generalized functions f that satisfy each of the following equations:
- (a)[10pts] $(x^2 - 1) \cdot f(x) = 0$;
 - (b)[10pts] $(x^4 - 1) \cdot f(x) = \delta(x)$.
61. [15pts] Compute the derivative in the distribution (or, weak) sense of the function

$$f(x) = \begin{cases} x^2 - 2 & \text{if } x < 0, \\ \sin(x) & \text{if } 0 \leq x < \pi/2, \\ 2 \cos(x) + x & \text{if } x \geq \pi/2. \end{cases}$$

62. [20pts] Find all generalized functions f that satisfy each of the following (inhomogeneous) equations.
- (a)[10pts] $(D^4 - 1)f(x) = \delta(x)$; D is the differential operator $\frac{d}{dx}$, thus $D^4 f(x) = \frac{d^4 f}{dx^4}(x)$.
Hint: Fourier transform the equation, solve an algebraic equation, and then invert.
 - (b)[10pts] $(D^2 - 1)f(x) = \cos(x)$.
Hint: Either use convolution with the Green's function; or Fourier transform this equation.