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THE LOW-FREQUENCY ELECTRIC FIELDS INDUCED IN A SPHERICAL CELL INCLUDING ITS NUCLEUS

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Abstract—After a review of work leading to the determination of the electric field induced in the human body when exposed to the electromagnetic field near an extremely-low-frequency high-voltage transmission line, attention is directed to a spherical cell exposed to the electric field in the body. Following a brief discussion of the potential biological significance of the study, the electric field acting on the surface of the cell and the electric field induced by it in the region between the outer cell membrane and the nuclear envelope are determined analytically, as is the field induced in the nucleus. It is shown, as an example, that when the body is exposed to a 60-Hz axial electric field of 2100 V/m, the field induced in the nucleus of a cell in the body is 0.27 nV/m when the radius of the nucleus is half that of the cell. The biologically interesting electric field along the outer surface of the nuclear envelope is in this case 2.7 μV/m. The simple analytical formulas can be applied to other values of the parameters, such as varying sizes of the nucleus (see Figure 2), and to power-line fields of different magnitude.

1. INTRODUCTION

When the human body is exposed to the electromagnetic fields of a high-voltage transmission line, electric currents and fields are induced in the body since at extremely low frequencies (ELF) (50–60 Hz) all the organs in the body are conductors. Are these currents and fields causes of or contributors to the incidence of malignancies? Answers to these questions require the cooperation of applied physicists and biomedical scientists. The physicists' contributions are (1) the accurate determination of the electromagnetic fields in houses and yards located near a typical three-wire, three-phase high-voltage transmission line, and (2) the derivation of accurate formulas for the electric currents and fields induced in the organs and cells in the body. Much of this work has been completed including the determination of the electric field in the membrane and interior of spherical and cylindrical cells. However, the cells studied either have no nucleus or the nucleus is assumed to be uncoupled to the exterior membrane of the cell. It is the purpose of this paper to determine the complete electric field in all parts of a spherical cell with a nucleus of arbitrary size, including the interaction between the nucleus and the cell membrane. It is biologically important to determine the correct electric field in and near the nucleus as shown by the following example.

In a recent article in *Science*, E. Pennisi [1] begins with the statement: "At the turn of the century, a European biologist named Theodor Boveri suggested that a small body near the cell nucleus might be a key to cancer. Called the centrosome, it replicates before cell division and then, via the protein cables that radiate from it, helps pull the duplicated chromosomes apart into the daughter cells. Boveri proposed that errors in this process could derange cells." Pennisi then refers to a paper by Hinchcliffe et al. [2] in the same issue of *Science* which reports the identification of a trigger that helps to tell the dividing cell to copy the centrosome, viz., the enzyme Cdk2. Pennisi continues with the statement: "By helping explain how the cell cycle and centrosome replication are linked so that the centrosome is copied just once and at just the right time, the finding may help researchers figure out how the replication might go awry, with potentially disastrous consequences." Can such consequences be caused or stimulated by an induced ELF electric field? Clearly, a meaningful answer to this question requires knowledge of the accurate electric field induced near the nucleus of the cell. Other initiatives looking for answers to the cause of cancer necessarily involve the nucleus, its envelope, and the region near it.

2. RECENT BACKGROUND

Extensive data on electromagnetic fields in the human body and on the electrical properties of biological substances are in articles by O. P. Gandhi and M. A. Stuchly in the book by Gandhi [3]. More recently, a study has been carried out at Harvard University to determine analytically the electric field induced in the human body and in a cell within the body when exposed to the electromagnetic field of a high-voltage transmission line. The first step was to provide formulas and tables for the direction and magnitude of these fields. This was carried out by King and Wu [4] and Margetis [5–7]. A survey of houses near such lines showed that some are as close as 10 to 30 m where the electric field may be of the order of 2100 V/m. The second step was to evaluate the electric field in houses near a power line [8]. The third step was to determine currents and electromagnetic fields induced in the human body. This was carried out by King and Wu [4], who showed that the electric field parallel to the length of the body induces the largest field in the body. The circulating electric currents and fields induced by the magnetic field are negligibly small in comparison. The electric fields induced in the different organs of the body were determined by King and Sandler [9]. These studies assumed that the arms are in contact with the body. Generalization to determine the electric field induced in the body, arms, legs, and head when the arms are raised was carried out by King [10], who showed that the electric field induced in the body when exposed to an axial field of 2140 V/m is of the order of 4 mV/m. This is the electric field in the saline fluid which permeates most of the body and in which cells are embedded. The analysis of a small spherical cell has been carried out by Foster and Schwan [11] who studied the spherical cell with a nucleus but assumed that the nucleus is so small compared to the cell that the electric field incident on the nucleus is the same as the field in the absence of the nucleus. Under these assumptions, they provide a formula for the field in the nucleus. Since this does not take into account the interaction between the nucleus and the outer membrane, it cannot give the correct field near or in the nucleus. King [12] has provided complete formulas for the electric field induced in the interior and in the membrane of a spherical cell without nucleus. The complete electric field induced in all parts of a spherical cell with nucleus does not appear to have been determined and is the subject of this investigation, specifically at ELF frequencies.

3. DIRECT ANALYSIS

An exact analysis can be made of the electric field induced in both the nucleus and the surrounding interior of a cell embedded in saline fluid if certain geometrical and biological simplifications are made. The calculated induced electric field is nonetheless accurately given. Specifically, the cell is assumed to be spherical with a concentric nucleus. All surfaces are smooth. Actually the surfaces of membranes have irregular protuberances and depressions [13]. However, use of the average thickness results in an average induced electric field that corresponds to that obtained with the simplified form.

Let the inner radius of the nucleus (region 1) be a and the outer radius be $b = a + \delta_2$, where δ_2 is the thickness of the nuclear envelope (region 2). The inner radius of the cell membrane is $c = 10^{-6}$ m (region 3), and its outer radius is $d = c + \delta_4$, where δ_4 is the thickness of the cell membrane. The tissue outside the cell is region 5 and the electric field incident on the cell is E_{5z}^{inc}. The radius b of the nucleus with its envelope can be assigned any appropriate value. For convenience, the ratio b/c is set equal to n. In general, $n \leq 0.5$. In the numerical calculations in this paper, it will be given the value $n = 0.5$. The nuclear envelope consists of two concentric membranes, each with the thickness 2.5×10^{-9} m, and separated by a distance 5×10^{-8} m. The conductivity of the membranes is $\sigma_2 \approx 10^{-6}$ S/m, and that of the region between them is $\sigma \approx 0.5$ S/m. To simplify the analysis, the entire nuclear envelope is represented by a homogeneous region 2 with the thickness $\delta_2 = 5 \times 10^{-8}$ m and conductivity $\sigma_2 = 5 \times 10^{-6}$ S/m. This gives the same induced field inside and outside the envelope as does the actual envelope with the two membranes. The conductivity of the interior of the nucleus is $\sigma_1 = 0.5$ S/m, that of the cell between the nuclear envelope and the cell membrane is $\sigma_3 = 0.5$ S/m. Let $\epsilon_2 = \sigma_2/\sigma_1 = 5 \times 10^{-6}/0.5 = 10^{-5}$; $\epsilon_4 = \sigma_4/\sigma_3 = 10^{-6}/0.5 = 2 \times 10^{-6}$. A schematic diagram of the cell and list of relevant dimensions are in Figure 1.

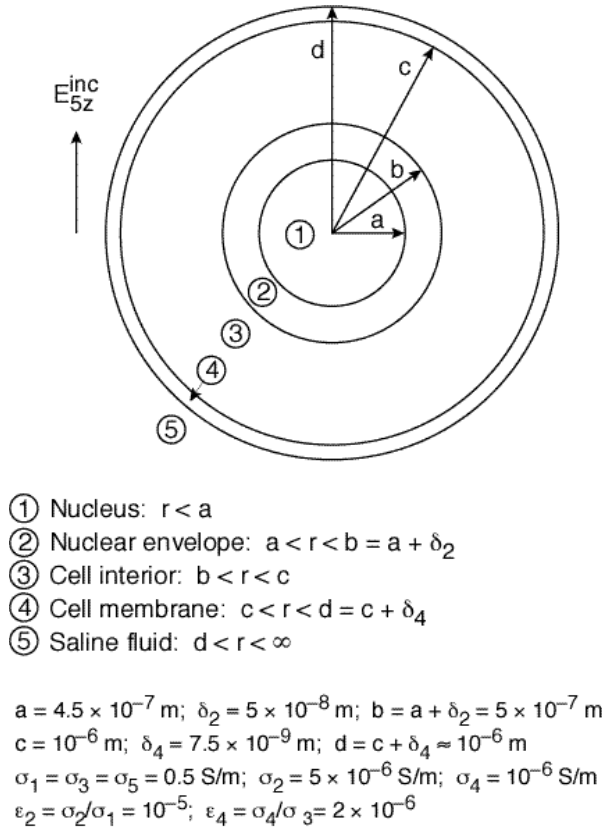

Figure 1. Spherical cell with nucleus exposed to incident field E_{5z}^{inc}.

The boundary conditions between region 1 ($r < a$) and region 2 ($a < r < b$), region 2 and region 3 ($b < r < c$), region 3 and region 4 ($c < r < d$), and region 4 and region 5 ($r > d$) are:

$$\begin{aligned}
r = a&: & E_{1r} &= \epsilon_2 E_{2r}, & \phi_1 &= \phi_2,\\
r = b&: & \epsilon_2 E_{2r} &= E_{3r}, & \phi_2 &= \phi_3,\\
r = c&: & E_{3r} &= \epsilon_4 E_{4r}, & \phi_3 &= \phi_4,\\
r = d&: & \epsilon_4 E_{4r} &= E_{5r}, & \phi_4 &= \phi_5.
\end{aligned} \tag{1}$$

Since the cell is rotationally symmetric and the incident field E_{5z}^{inc} is constant over the dimensions of the cell, spherical coordinates can be used and

$$\phi \equiv \phi(r, \theta) = f(r) \cos\theta. \tag{2}$$

The scalar potential satisfies the differential equation

$$\nabla^2\phi(r,\theta) = \cos\theta\left(\frac{\partial^2 f}{\partial r^2} + \frac{2}{r}\frac{\partial f}{\partial r} - \frac{2}{r}f\right) = 0. \tag{3}$$

This has the general solution

$$f = f(r) = C_1 r + C_2/r^2, \tag{4}$$

where C_1 and C_2 are constants to be determined from the boundary conditions (1) and $f(r) \to -rE_{5z}^{\rm inc}$ for $r \gg d$.

The solution for $\phi(r,\theta) = f(r)\cos\theta$ in the five regions requires that $f(r)$ have the following forms:

$$\begin{aligned}
&f_1(r) = A_1 r, && r < a,\\
&f_2(r) = A_2' r + A_2''/r^2, && a < r < b,\\
&f_3(r) = A_3' r + A_3''/r^2, && b < r < c,\\
&f_4(r) = A_4' r + A_4''/r^2, && c < r < d,\\
&f_5(r) = -rE_{5z}^{\rm inc} + A_5/r^2, && r > d.
\end{aligned} \tag{5}$$

The conditions for determining the eight constants A are obtained from (1)–(4), viz.,

$$\begin{aligned}
&r = a\colon && f_1(a) = f_2(a), && \partial f_1(r)/\partial r = \epsilon_2[\partial f_2(r)/\partial r],\\
&r = b\colon && f_2(b) = f_3(b), && \epsilon_2[\partial f_2(r)/\partial r] = \partial f_3(r)/\partial r,\\
&r = c\colon && f_3(c) = f_4(c), && \partial f_3(r)/\partial r = \epsilon_4[\partial f_4(r)/\partial r],\\
&r = d\colon && f_4(d) = f_5(d), && \epsilon_4[\partial f_4(r)/\partial r] = \partial f_5(r)/\partial r.
\end{aligned} \tag{6}$$

With $f_j(r)$, $j = 1$–5, given by (5), the eight relations in (6) are

$$\begin{aligned}
r = a: && A_1 &= A_2' + A_2''/a^3,\\
&& A_1 &= \epsilon_2[A_2' - 2A_2''/a^3],\\
r = b: && A_2' + A_2''/b^3 &= A_3' + A_3''/b^3,\\
&& \epsilon_2[A_2' - 2A_2''/b^3] &= A_3' - 2A_3''/b^3,\\
r = c: && A_3' + A_3''/c^3 &= A_4' + A_4''/c^3,\\
&& A_3' - 2A_3''/c^3 &= \epsilon_4[A_4' - 2A_4''/c^3],\\
r = d: && A_4' + A_4''/d^3 &= -E_{5z}^{\rm inc} + A_5/d^3,\\
&& \epsilon_4[A_4' - 2A_4''/d^3] &= -E_{5z}^{\rm inc} - 2A_5/d^3.
\end{aligned} \tag{7}$$

The set of equations to be solved is

$$
\begin{aligned}
A_1 - A_2' - A_2''/a^3 &= 0, \\
A_1 - \epsilon_2 A_2' + 2\epsilon_2 A_2''/a^3 &= 0, \\
A_2' + A_2''/b^3 - A_3' - A_3''/b^3 &= 0, \\
\epsilon_2 A_2' - 2\epsilon_2 A_2''/b^3 - A_3' + 2A_3''/b^3 &= 0, \\
A_3' + A_3''/c^3 - A_4' - A_4''/c^3 &= 0, \\
A_3' - 2A_3''/c^3 - \epsilon_4 A_4' + 2\epsilon_4 A_4''/c^3 &= 0, \\
A_4' + A_4''/d^3 - A_5/d^3 &= -E_{5z}^{\mathrm{inc}}, \\
\epsilon_4 A_4' - 2\epsilon_4 A_4''/d^3 + 2A_5/d^3 &= -E_{5z}^{\mathrm{inc}}.
\end{aligned}
\tag{8}
$$

The determinant of the coefficients is

$$
D \equiv \begin{vmatrix}
1 & -1 & -1/a^3 & 0 & 0 & 0 & 0 & 0 \\
1 & -\epsilon_2 & 2\epsilon_2/a^3 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1/b^3 & -1 & -1/b^3 & 0 & 0 & 0 \\
0 & \epsilon_2 & -2\epsilon_2/b^3 & -1 & 2/b^3 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1/c^3 & -1 & -1/c^3 & 0 \\
0 & 0 & 0 & 1 & -2/c^3 & -\epsilon_4 & 2\epsilon_4/c^3 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1/d^3 & -1/d^3 \\
0 & 0 & 0 & 0 & 0 & \epsilon_4 & -2\epsilon_4/d^3 & 2/d^3
\end{vmatrix}.
\tag{9}
$$

The solution for the electric field induced in the interior of the nucleus is

$$
A_1 = D^{-1} N_1, \tag{10}
$$

where

$$
N_1 \equiv \begin{vmatrix}
0 & -1 & -1/a^3 & 0 & 0 & 0 & 0 & 0 \\
0 & -\epsilon_2 & 2\epsilon_2/a^3 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1/b^3 & -1 & -1/b^3 & 0 & 0 & 0 \\
0 & \epsilon_2 & -2\epsilon_2/b^3 & -1 & 2/b^3 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1/c^3 & -1 & -1/c^3 & 0 \\
0 & 0 & 0 & 1 & -2/c^3 & -\epsilon_4 & 2\epsilon_4/c^3 & 0 \\
-E_{5z}^{\mathrm{inc}} & 0 & 0 & 0 & 0 & 1 & 1/d^3 & -1/d^3 \\
-E_{5z}^{\mathrm{inc}} & 0 & 0 & 0 & 0 & \epsilon_4 & -2\epsilon_4/d^3 & 2/d^3
\end{vmatrix}.
\tag{11}
$$

With A_1 determined, the potential in the nucleus is

$$
\phi_1(r, \theta) = f_1(r) \cos\theta = A_1 r \cos\theta, \tag{12}
$$

and the electric field is

$$\begin{aligned}\mathbf{E}_1 &= -\nabla\phi_1(r,\theta) = -\left(\hat{\mathbf{r}}\,\frac{\partial\phi_1}{\partial r} + \hat{\boldsymbol{\theta}}\,\frac{1}{r}\,\frac{\partial\phi_1}{\partial\theta}\right)\\ &= -A_1[\hat{\mathbf{r}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta] = -A_1\hat{\mathbf{z}}.\end{aligned} \tag{13}$$

The electric field in the cell between the nuclear envelope and the outer membrane requires the evaluation of A_3' and A_3'', where

$$A_3' = D^{-1}N_3', \quad A_3'' = D^{-1}N_3'', \tag{14}$$

and N_3' and N_3'' are obtained from D by replacing, respectively, columns 4 and 5 in (9) by column 1 in (11). The potential in region 3 is

$$\phi_3(r,\theta) = f_3(r)\cos\theta = (A_3'r + A_3''/r^2)\cos\theta, \tag{15}$$

and the electric field is

$$\begin{aligned}\mathbf{E}_3 &= -\nabla\phi_3(r,\theta) = -\left(\hat{\mathbf{r}}\,\frac{\partial\phi_3}{\partial r} + \hat{\boldsymbol{\theta}}\,\frac{1}{r}\,\frac{\partial\phi_3}{\partial\theta}\right)\\ &= -\left[\hat{\mathbf{r}}(A_3' - 2A_3''/r^3)\cos\theta - \hat{\boldsymbol{\theta}}(A_3' + A_3''/r^3)\sin\theta\right].\end{aligned} \tag{16}$$

The complete formula for $\mathbf{E}_1$ is given by (13) with (10) substituted for A_1. The 8×8 matrix that must be inverted to solve the problem numerically consists of values $O(10^{-6})$ through 1.0 to $O(10^{18})$, with the smallest appearing at times on the diagonal. Such a matrix is ill-conditioned and numerically very close to singular. The set of techniques known as singular value decomposition can solve this kind of inversion problem [14]. The estimate of the rank of the matrix produced as a by-product of the inversion is 5 or less, indicating that the inverse does not exactly solve $DD^{-1} = I$, but will be very close in the least squares sense. The solution in this case was not close enough to produce a physically reasonable result.

4. SEQUENTIAL ANALYSIS

Fortunately the basic equations for the eight coefficients A permit an alternative analytical approach that leads to exact formulas in a form that permits a complete determination of the eight coefficients. By solving each pair of equations in (8) for A_j in terms of A_{j+1}, the following formulas are obtained:

$$A_1 = \frac{3\epsilon_2}{1+2\epsilon_2}\,A_2',$$

$$A_1 = -\frac{3\epsilon_2}{1-\epsilon_2}\frac{1}{a^3}A_2'', \tag{17}$$

$$A_2' = \frac{1+2\epsilon_2}{3\epsilon_2}A_3' - \frac{2(1-\epsilon_2)}{3\epsilon_2}\frac{1}{b^3}A_3'',$$

$$A_2'' = -\frac{1-\epsilon_2}{3\epsilon_2}b^3A_3' + \frac{2+\epsilon_2}{3\epsilon_2}A_3'', \tag{18}$$

$$A_3' = \frac{2+\epsilon_4}{3}A_4' + \frac{2(1-\epsilon_4)}{3}\frac{1}{c^3}A_4'',$$

$$A_3'' = \frac{1-\epsilon_4}{3}c^3A_4' + \frac{1+2\epsilon_4}{3}A_4'', \tag{19}$$

$$A_4' = -\frac{1+2\epsilon_4}{3\epsilon_4}E_{5z}^{\text{inc}} - \frac{2(1-\epsilon_4)}{3\epsilon_4}\frac{1}{d^3}A_5,$$

$$A_4'' = \frac{1-\epsilon_4}{3\epsilon_4}d^3E_{5z}^{\text{inc}} + \frac{2+\epsilon_4}{3\epsilon_4}A_5. \tag{20}$$

From these formulas, two different expressions can be obtained for A_1, in terms of A_5 and E_{5z}^{inc}. This is accomplished by successively substituting (20) into (19), (19) into (18), and (18) into (17). This yields two different expressions for A_1. By equating these, A_5 can be obtained in terms of E_{5z}^{inc}. When this is substituted in either formula for A_1, A_1 is obtained in terms of E_{5z}^{inc}. This is the desired solution. The entire procedure is carried out systematically in the Appendix. With A_1 determined, it can be substituted in (18) to obtain A_2' and A_2''. Continuing substitutions yield A_3' and A_3'', A_4' and A_4''. Thus, all eight coefficients A are available in explicit form. Their determination is carried out in the Appendix where explicit general formulas are given.

4.1. The Electric Fields in and Near the Nucleus

Of special interest in this paper is the electric field induced in the nucleus (region 1, $r < a$) and between the nucleus and the outer cell membrane (region 3, $b < r < c$). As given in the Appendix, the formula for A_1 is

$$A_1 = 81\epsilon_2\epsilon_4\mathcal{D}^{-1}E_{5z}^{\text{inc}}, \tag{21}$$

where

$$\begin{aligned}\mathcal{D} = &-[(1+2\epsilon_2)(2+\epsilon_2) - 2(1-\epsilon_2)^2a^3/b^3] \\ &\times [(1+2\epsilon_4)(2+\epsilon_4) - 2(1-\epsilon_4)^2c^3/d^3] \\ &+ 2(b^3/c^3)(1-\epsilon_2)(1+2\epsilon_2)(1-\epsilon_4)(2+\epsilon_4)(1-a^3/b^3)(1-c^3/d^3).\end{aligned} \tag{22}$$

It is readily verified by setting $\epsilon_2 = 1$ that (21) with (22) reduces to formula (14) of King [12] for the cell without nucleus.

The expression for $\mathcal{D}$ can be greatly simplified if the thickness δ_2 of the nuclear envelope and δ_4 of the cell membrane are introduced since quantities of the order ϵ_2^2, ϵ_4^2, and $\epsilon_2\epsilon_4$ are negligible compared to 1. Specifically, $(1+2\epsilon_{2,4})(2+\epsilon_{2,4}) \approx 2+5\epsilon_{2,4}$, $(1-\epsilon_{2,4})^2 \approx 1-2\epsilon_{2,4}$, and $(1-\epsilon_4)(2+\epsilon_4)(1-\epsilon_2)(1+2\epsilon_2) \approx 2+2\epsilon_2-\epsilon_4$. Also, $1-c^3/d^3 = 1-(d-\delta_4)^3/d^3 \approx 1-(1-\delta_4/d)^3 \approx 1-(1-3\delta_4/d) = 3\delta_4/d$; similarly, $1-a^3/b^3 \approx 3\delta_2/b$. With these substitutions,

$$\begin{aligned}
\mathcal{D} \;&\approx\; -\Big[2+5\epsilon_2-(2-4\epsilon_2)a^3/b^3\Big]\Big[2+5\epsilon_4-(2-4\epsilon_4)c^3/d^3\Big] \\
&\quad + 2(b^3/c^3)(2+2\epsilon_2-\epsilon_4)(3\delta_2/b)(3\delta_4/d) \\
&\approx\; -\Big[2(1-a^3/b^3)+9\epsilon_2\Big]\Big[2(1-c^3/d^3)+9\epsilon_4\Big] \\
&\quad + 4(b^3/c^3)(3\delta_2/b)(3\delta_4/d) \\
&\approx\; -\big\{4(3\delta_2/b)(3\delta_4/d)+18[\epsilon_2(3\delta_4/d)+\epsilon_4(3\delta_2/b)] \\
&\quad - 4(b^3/c^3)(3\delta_2/b)(3\delta_4/d)\big\}. \qquad (23)
\end{aligned}$$

The leading terms are

$$\begin{aligned}
\mathcal{D} \;&\approx\; -4(1-b^3/c^3)(3\delta_2/b)(3\delta_4/d) \\
&=\; -4(1-n^3)(3\delta_2/b)(3\delta_4/d), \qquad (24)
\end{aligned}$$

where $n = b/c$. Hence,

$$\begin{aligned}
A_1 \;&\approx\; -\frac{81\epsilon_2\epsilon_4 E_{5z}^{\mathrm{inc}}}{4(1-n^3)(3\delta_2/b)(3\delta_4/d)} \\
&=\; -(1-n^3)^{-1}(3\epsilon_2 b/2\delta_2)(3\epsilon_4 d/2\delta_4)E_{5z}^{\mathrm{inc}}. \qquad (25)
\end{aligned}$$

For convenience, let

$$\beta \equiv (3\epsilon_2 b/2\delta_2), \quad \Delta \equiv (3\epsilon_4 d/2\delta_4),$$

so that

$$A_1 \approx -(1-n^3)^{-1}\beta\Delta\, E_{5z}^{\mathrm{inc}}. \qquad (26)$$

With $\epsilon_2 = 10^{-5}$, $\epsilon_4 = 2\times10^{-6}$, $\delta_2 = 5\times10^{-8}$ m, $\delta_4 = 7.5\times10^{-9}$ m, $b = 5\times10^{-7}$ m, $c = 10^{-6}$ m, $n = b/c = 0.5$, and $d \approx 10^{-6}$ m, it follows that

$$\beta = 1.5\times10^{-4}, \quad \Delta = 4\times10^{-4},$$

and

$$A_1 \;=\; -\frac{6\times10^{-8}}{0.875}\,E_{5z}^{\mathrm{inc}} = -6.86\times10^{-8}E_{5z}^{\mathrm{inc}}. \qquad (27)$$

When A_1 is substituted in (13), the electric field induced in the nucleus is

$$\mathbf{E}_1 = -\hat{\mathbf{z}}A_1 \approx \hat{\mathbf{z}}\, 0.686 \times 10^{-7} E_{5z}^{\text{inc}}. \tag{28a}$$

Alternatively,

$$\mathbf{E}_1 \approx 0.686(\hat{\mathbf{r}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta) \times 10^{-7} E_{5z}^{\text{inc}}. \tag{28b}$$

The electric field induced in region 3 ($b < r < c$) between the nucleus and the outer cell membrane is given by (16). It requires the evaluation of A_3' and A_3''. The formulas obtained from the Appendix are

$$\begin{aligned} A_3' &\approx \frac{9\epsilon_4[2(3\delta_2/b) + 9\epsilon_2]E_{5z}^{\text{inc}}}{-4(1-n^3)(3\delta_2/b)(3\delta_4/d)} \\ &= -\left[4.5\left(\frac{\epsilon_4 d}{3\delta_4}\right) + \frac{81}{4}\left(\frac{\epsilon_2 b}{3\delta_2}\right)\left(\frac{\epsilon_4 d}{3\delta_4}\right)\right](1-n^3)^{-1}E_{5z}^{\text{inc}} \\ &= -\Delta(1+\beta)(1-n^3)^{-1}E_{5z}^{\text{inc}}, \end{aligned} \tag{29}$$

$$\begin{aligned} A_3'' &\approx \frac{9\epsilon_4(3\delta_2/b)b^3E_{5z}^{\text{inc}}}{-4(1-n^3)(3\delta_2/b)(3\delta_4/d)} = -\frac{4.5}{2}\left(\frac{\epsilon_4 d}{3\delta_4}\right)b^3(1-n^3)^{-1}E_{5z}^{\text{inc}} \\ &= -0.5b^3\Delta(1-n^3)^{-1}E_{5z}^{\text{inc}}. \end{aligned} \tag{30}$$

It follows that

$$\begin{aligned} \mathbf{E}_3 &= -\left[\hat{\mathbf{r}}(A_3' - 2A_3''/r^3)\cos\theta - \hat{\boldsymbol{\theta}}(A_3' + A_3''/r^3)\sin\theta\right] \\ &\approx E_{5z}^{\text{inc}}\Delta(1-n^3)^{-1}\left[\hat{\mathbf{r}}\left(1+\beta-\frac{b^3}{r^3}\right)\cos\theta - \hat{\boldsymbol{\theta}}\left(1+\frac{b^3}{2r^3}\right)\sin\theta\right]. \end{aligned} \tag{31}$$

In the last term in (31), $\beta = 1.5 \times 10^{-4}$ is negligible and has been omitted. At $r = c$,

$$\begin{aligned} \mathbf{E}_3 &\approx E_{5z}^{\text{inc}}\Delta(1-n^3)^{-1}\left[\hat{\mathbf{r}}(1+\beta-n^3)\cos\theta - \hat{\boldsymbol{\theta}}(1+0.5n^3)\sin\theta\right] \\ &\approx E_{5z}^{\text{inc}}\Delta\left[\hat{\mathbf{r}}\cos\theta - \hat{\boldsymbol{\theta}}\left(\frac{2+n^3}{2(1-n^3)}\right)\sin\theta\right]. \end{aligned} \tag{32}$$

At $r = b$,

$$\mathbf{E}_3 \approx E_{5z}^{\text{inc}}\Delta(1-n^3)^{-1}\left[\hat{\mathbf{r}}\beta\cos\theta - 1.5\hat{\boldsymbol{\theta}}\sin\theta\right] \tag{33a}$$

$$= E_{5z}^{\text{inc}}(1-n^3)^{-1}\left[6\times 10^{-8}\hat{\mathbf{r}}\cos\theta - 6\times 10^{-4}\hat{\boldsymbol{\theta}}\sin\theta\right]. \tag{33b}$$

The electric field that enters the nuclear envelope perpendicularly and continues into the nucleus is

$$E_{3r} \approx E_{5z}^{\mathrm{inc}}(1-n^3)^{-1} \times 6 \times 10^{-8} \cos\theta. \tag{34}$$

This agrees with E_{1r} in (28b).

4.2. The Electric Field Acting on the Surface of the Cell

The final step in the analysis is to determine the normal component of the electric field on the outside surface of the cell. It is this field that crosses the cell membrane and enters the cells. It is given by the first term in

$$\mathbf{E}_5 = -\left[\hat{\mathbf{r}}\,\frac{\partial f_5}{\partial r}\,\cos\theta - \hat{\boldsymbol{\theta}}\,\frac{f_5}{r}\,\sin\theta\right],$$

where $f_5 = -rE_{5z}^{\mathrm{inc}} + A_5/r^2$. With this

$$\mathbf{E}_5 = \hat{\mathbf{r}}\left[E_{5z}^{\mathrm{inc}} + \frac{2}{r^3}\,A_5\right]\cos\theta - \hat{\boldsymbol{\theta}}\left[E_{5z}^{\mathrm{inc}} - \frac{1}{r^3}\,A_5\right]\sin\theta. \tag{35}$$

Here, $A_5 = \mathcal{N}\mathcal{D}^{-1}E_{5z}^{\mathrm{inc}}$, where the complete formula for $\mathcal{N}$ is given in the Appendix. It can be reduced to

$$\mathcal{N} \approx 2d^3\left[(1-n^3)(3\delta_2/b)(3\delta_4/d) + 4.5\epsilon_2(3\delta_4/d) + 4.5n^3\epsilon_4(3\delta_2/b)\right]. \tag{36}$$

$\mathcal{D}$ is given in (23). Rearrangement of the terms gives

$$\mathcal{D} \approx -4\left\{(1-n^3)(3\delta_2/b)(3\delta_4/d) + 4.5\left[\epsilon_2(3\delta_4/d) + \epsilon_4(3\delta_2/b)\right]\right\}. \tag{37}$$

It follows that

$$\frac{\mathcal{N}}{\mathcal{D}} \approx -\frac{d^3}{2}\left\{\frac{(1-n^3)(3\delta_2/b)(3\delta_4/d) + 4.5[\epsilon_2(3\delta_4/d) + n^3\epsilon_4(3\delta_2/b)]}{(1-n^3)(3\delta_2/b)(3\delta_4/d) + 4.5[\epsilon_2(3\delta_4/d) + \epsilon_4(3\delta_2/b)]}\right\}. \tag{38}$$

This can be rearranged to give

$$\frac{\mathcal{N}}{\mathcal{D}} \approx -\frac{d^3}{2}\left[1-n^3+\frac{3\epsilon_2 b}{2\delta_2}+n^3\left(\frac{3\epsilon_4 d}{2\delta_4}\right)\right]\left[1-n^3+\frac{3\epsilon_2 b}{2\delta_2}+\frac{3\epsilon_4 d}{2\delta_4}\right]^{-1}. \tag{39a}$$

With $\beta \equiv 3\epsilon_2 b/2\delta_2$ and $\Delta \equiv 3\epsilon_4 d/2\delta_4$, this becomes

$$\frac{\mathcal{N}}{\mathcal{D}} \approx -\frac{d^3}{2}\left[1+(1-n^3)^{-1}(\beta+n^3\Delta)\right]\left[1+(1-n^3)^{-1}(\beta+\Delta)\right]^{-1}. \tag{39b}$$

Since $\beta \ll 1$ and $\Delta \ll 1$,

$$\frac{\mathcal{N}}{\mathcal{D}} \approx -\frac{d^3}{2}\left\{1-(1-n^3)^{-1}[(\beta+\Delta)-(\beta+n^3\Delta)]\right\} = -\frac{d^3}{2}(1-\Delta). \quad (40)$$

Hence

$$A_5 \approx -\frac{d^3}{2}(1-\Delta)E_{5z}^{\text{inc}}. \quad (41)$$

Since this is the same as for the cell without nucleus, it is clear that the presence of a nucleus of any size has no observable effect on the electric field outside the cell. When (41) is substituted in (35), this becomes

$$\mathbf{E}_5 \approx E_{5z}^{\text{inc}}\left\{\hat{\mathbf{r}}\left[1-\frac{d^3}{r^3}(1-\Delta)\right]\cos\theta - \hat{\boldsymbol{\theta}}\left[1+\frac{d^3}{2r^3}\right]\sin\theta\right\}. \quad (42)$$

When $r = d$ on the outside surface of the cell,

$$\mathbf{E}_5 \approx E_{5z}^{\text{inc}}[\hat{\mathbf{r}}\Delta\cos\theta - 1.5\hat{\boldsymbol{\theta}}\sin\theta]. \quad (43)$$

The radial field that crosses the membrane and enters the cell is

$$E_{5r} \approx E_{5z}^{\text{inc}}\Delta\cos\theta = E_{5z}^{\text{inc}} \times 4 \times 10^{-4}\cos\theta. \quad (44)$$

4.3. The Physical Picture

An interesting and useful review of the electric field induced in a cell in the human body is in terms of the electric currents. These are given by the current density. Far from the cell, this is $J_{5z}^{\text{inc}} = \sigma E_{5z}^{\text{inc}}$. This is the alternating current through each unit volume of the saline fluid in the body. When this current impinges on a cell, most of it flows around the cell. The circulating current is obtained from (42). It is

$$J_{5\theta}(d) \approx -1.5J_{5z}^{\text{inc}}\sin\theta. \quad (45)$$

The small current that flows through the membrane is

$$J_{5r}(d) \approx J_{5z}^{\text{inc}}\Delta\cos\theta, \quad (46)$$

where $\Delta = (3\epsilon_4 d/2\delta_4) = 4\times10^{-4}$. Note that $J_{5\theta}$ on the surface of the cell is greater than J_{5z}^{inc} far from the cell by a factor 1.5. This is a consequence of the fact that the total current $I_z = \pi d^2 J_{5z}^{\text{inc}}$ that would have flowed in the saline fluid in the absence of the cell is forced to diverge around the cell with an increased density.

The current density in region 3 between the membrane and the nuclear envelope is obtained from (31). It is

$$\mathbf{J}_3(r) \approx J_{5z}^{\text{inc}} \Delta (1-n^3)^{-1} \left\{ \hat{\mathbf{r}} \left[1 + \beta - \frac{b^3}{r^3} \right] \cos\theta - \hat{\boldsymbol{\theta}} \left(1 + \frac{b^3}{2r^3} \right) \sin\theta \right\}, \tag{47}$$

where $\beta = (3\epsilon_2 b / 2\delta_2) = 1.5 \times 10^{-4}$ and $n = b/c$. The density of current entering the cell when $r = c$ is obtained from (32) or from (47). It is $J_{3r}(c) \approx J_{5z}^{\text{inc}} \Delta \cos\theta$ in agreement with (46). A circulating current also exists on the inner surface due to reflections from the nucleus. Its density is given by

$$J_{3\theta}(c) \approx -J_{5z}^{\text{inc}} \Delta \left(\frac{2 + n^3}{2(1-n^3)} \right) \sin\theta.$$

Note that when the nucleus is small enough so that $n^3 \ll 1$, $J_{3\theta}(c) \approx -J_{5z}^{\text{inc}} \Delta \sin\theta$. This is true when $n \leq 0.5$. The current density on the outer surface of the nuclear membrane is given by (47) with $r = b$. It is

$$\mathbf{J}_3(b) \approx J_{5z}^{\text{inc}} \Delta (1-n^3)^{-1} [\hat{\mathbf{r}} \beta \cos\theta - 1.5 \hat{\boldsymbol{\theta}} \sin\theta]. \tag{48}$$

The radial current into the membrane has the density $J_{3r} \approx J_{5z}^{\text{inc}} \beta \Delta (1 - n^3)^{-1} \cos\theta$, where $\beta\Delta = 6 \times 10^{-8}$. Finally, the current density in the nucleus is obtained from (13) with (27). It is

$$\mathbf{J}_1 \approx J_{5z}^{\text{inc}} \beta \Delta (1-n^3)^{-1} [\hat{\mathbf{r}} \cos\theta - \hat{\boldsymbol{\theta}} \sin\theta] = \hat{\mathbf{z}} J_{5z}^{\text{inc}} \beta \Delta (1-n^3)^{-1}. \tag{49}$$

5. CONCLUSION

The electric field induced in all parts of a spherical cell when exposed to an incident ELF electric field E_{5z}^{inc} maintained in the saline fluid outside the cell has been derived. If the power-line electric field is 2100 V/m acting axially on a human being, the electric field in that body acting on a cell is $E_{5z}^{\text{inc}} = 4\,\text{mV/m}$. It follows that the field in the nucleus of that cell as given by (28a) is

$$\mathbf{E}_1 = \hat{\mathbf{z}} 0.686 \times 10^{-7} \times 4 \times 10^{-3}\ \text{V/m} = \hat{\mathbf{z}} 2.74 \times 10^{-10}\ \text{V/m} = \hat{\mathbf{z}} 0.274\ \text{nV/m}. \tag{50}$$

This is extremely small and presumably can have no deleterious effect on cell replication.

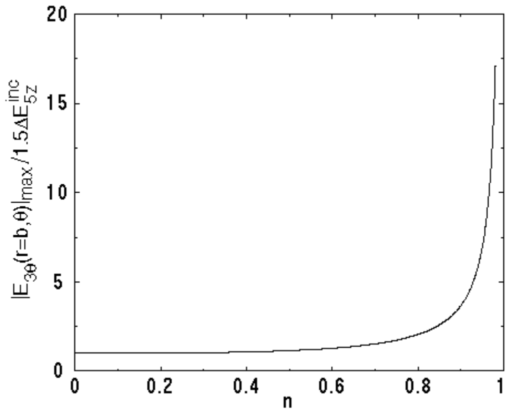

Figure 2. Maximum magnitude of tangential electric field at the outer surface of the nuclear envelope ($r = b,\ \theta = \pi/2$) as a function of relative nuclear size, $n = b/c \approx a/c$. Note that $\Delta = 3\epsilon_4 d/2\delta_4$ and E_{5z}^{inc} is the incident electric field.

The field near the surface of the nuclear envelope and acting parallel to its spherical surface is $E_{3\theta}$ in (33). It is

$$\begin{aligned} E_{3\theta} &= -6.86 \times 10^{-4} E_{5z}^{\text{inc}} \sin\theta \\ &= -6.86 \times 10^{-4} \times 4 \times 10^{-3} \sin\theta \text{ V/m} \\ &= -2.74 \sin\theta\ \mu\text{V/m}. \end{aligned} \tag{51}$$

This is a new result. The large tangential electric field on the outer surface of the nuclear envelope has not been determined previously. This is the field that should be of interest to those investigating the centrosome theory as a possible mechanism for the development of cancer. It is shown graphically in Figure 2 as a function of the relative size of the nucleus. It may also be of interest to those following other leads since the nucleus of cells is the location where the malignant multiplication of cells must have its inception. Are these electric fields determined for all locations in a cell significant in producing biological effects? Some studies seeking to provide an answer to this question have been published. Lee and McLeod [15] describe definite evidence of morphological adaptation of cells to ELF electric fields as low as 0.5 mV/m. Morphological changes in human Raji B lymphoid cells from a comparable exposure were also observed by Lisi et al. [16]. The ability of power-frequency electromagnetic fields to accelerate cell differentiation is described by Aaron et al. [17]. Also of interest is the study by Eibert et al. [18] on lipid bilayer membranes exposed to RF fields.

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APPENDIX A.

The system of equations (17)–(20) is a combination of the following 2×2 matrix equations:

$$\begin{bmatrix} a & 1/a^2 \\ \epsilon_2 & -2\epsilon_2/a^3 \end{bmatrix} \begin{bmatrix} A_2' \\ A_2'' \end{bmatrix} = A_1 \begin{bmatrix} a \\ 1 \end{bmatrix}, \tag{A1}$$

$$\begin{bmatrix} b & 1/b^2 \\ \epsilon_2 & -2\epsilon_2/b^3 \end{bmatrix} \begin{bmatrix} A_2' \\ A_2'' \end{bmatrix} = \begin{bmatrix} b & 1/b^2 \\ 1 & -2/b^3 \end{bmatrix} \begin{bmatrix} A_3' \\ A_3'' \end{bmatrix}, \tag{A2}$$

$$\begin{bmatrix} c & 1/c^2 \\ 1 & -2/c^3 \end{bmatrix} \begin{bmatrix} A_3' \\ A_3'' \end{bmatrix} = \begin{bmatrix} c & 1/c^2 \\ \epsilon_4 & -2\epsilon_4/c^3 \end{bmatrix} \begin{bmatrix} A_4' \\ A_4'' \end{bmatrix}, \tag{A3}$$

$$\begin{bmatrix} d & 1/d^2 \\ \epsilon_4 & -2\epsilon_4/d^3 \end{bmatrix} \begin{bmatrix} A_4' \\ A_4'' \end{bmatrix} = \begin{bmatrix} -d & 1/d^2 \\ -1 & -2/d^3 \end{bmatrix} \begin{bmatrix} E_{5z}^{\text{inc}} \\ A_5 \end{bmatrix}. \tag{A4}$$

Each of the 2×2 matrices appearing here is invertible. For example,

$$\begin{bmatrix} a & 1/a^2 \\ \epsilon_2 & -2\epsilon_2/a^3 \end{bmatrix}^{-1} = -\frac{a^2}{3\epsilon_2} \begin{bmatrix} -2\epsilon_2/a^3 & -1/a^2 \\ -\epsilon_2 & a \end{bmatrix}. \tag{A5}$$

Consequently,

$$\begin{aligned} \begin{bmatrix} E_{5z}^{\text{inc}} \\ A_5 \end{bmatrix} &= \begin{bmatrix} -d & 1/d^2 \\ -1 & -2/d^3 \end{bmatrix}^{-1} \begin{bmatrix} d & 1/d^2 \\ \epsilon_4 & -2\epsilon_4/d^3 \end{bmatrix} \begin{bmatrix} c & 1/c^2 \\ \epsilon_4 & -2\epsilon_4/c^3 \end{bmatrix}^{-1} \\ &\quad \times \begin{bmatrix} c & 1/c^2 \\ 1 & -2/c^3 \end{bmatrix} \begin{bmatrix} b & 1/b^2 \\ 1 & -2/b^3 \end{bmatrix}^{-1} \begin{bmatrix} b & 1/b^2 \\ \epsilon_2 & -2\epsilon_2/b^3 \end{bmatrix} \\ &\quad \times \begin{bmatrix} a & 1/a^2 \\ \epsilon_2 & -2\epsilon_2/a^3 \end{bmatrix}^{-1} \begin{bmatrix} a \\ 1 \end{bmatrix} A_1 \\ &= \frac{1}{3^4 \epsilon_2 \epsilon_4} \begin{bmatrix} \mathcal{D} \\ \mathcal{N} \end{bmatrix} A_1, \end{aligned} \tag{A6}$$

which yields

$$A_1 = 3^4 \epsilon_2 \epsilon_4 \mathcal{D}^{-1} E_{5z}^{\text{inc}}, \tag{A7}$$

$$A_5 = \mathcal{N} \mathcal{D}^{-1} E_{5z}^{\text{inc}}. \tag{A8}$$

In the above,

$$\begin{aligned}\mathcal{D} &= -[(1+2\epsilon_2)(2+\epsilon_2)-2(1-\epsilon_2)^2a^3/b^3] \\ &\quad\times[(1+2\epsilon_4)(2+\epsilon_4)-2(1-\epsilon_4)^2c^3/d^3] \\ &\quad+2(b^3/c^3)(1+2\epsilon_2)(1-\epsilon_2) \\ &\quad\times(2+\epsilon_4)(1-\epsilon_4)(1-a^3/b^3)(1-c^3/d^3), \qquad \text{(A9)}\end{aligned}$$

$$\begin{aligned}\mathcal{N} &= d^3[(1+2\epsilon_2)(2+\epsilon_2)-2(1-\epsilon_2)^2a^3/b^3] \\ &\quad\times(1+2\epsilon_4)(1-\epsilon_4)(1-c^3/d^3) \\ &\quad-b^3[2(1-\epsilon_4)^2d^3/c^3-(1+2\epsilon_4)(2+\epsilon_4)] \\ &\quad\times(1+2\epsilon_2)(1-\epsilon_2)(1-a^3/b^3). \qquad \text{(A10)}\end{aligned}$$

Accordingly, from (A1) and (A2),

$$\begin{aligned}A_2' &= 3^3\epsilon_4(1+2\epsilon_2)\mathcal{D}^{-1}E_{5z}^{\text{inc}}, && \text{(A11)}\\ A_2'' &= -3^3\epsilon_4(1-\epsilon_2)\mathcal{D}^{-1}a^3E_{5z}^{\text{inc}}, && \text{(A12)}\\ A_3' &= 3^2\epsilon_4[(1+2\epsilon_2)(2+\epsilon_2)-2(1-\epsilon_2)^2a^3/b^3]\mathcal{D}^{-1}E_{5z}^{\text{inc}}, && \text{(A13)}\\ A_3'' &= 3^2\epsilon_4(1+2\epsilon_2)(1-\epsilon_2)b^3(1-a^3/b^3)\mathcal{D}^{-1}E_{5z}^{\text{inc}}. && \text{(A14)}\end{aligned}$$

Finally, (A4) furnishes A_4' and A_4'':

$$A_4' = -\frac{1+2\epsilon_4+2(1-\epsilon_4)\mathcal{N}\mathcal{D}^{-1}/d^3}{3\epsilon_4}\,E_{5z}^{\text{inc}}, \qquad \text{(A15)}$$

$$A_4'' = \frac{1-\epsilon_4+(2+\epsilon_4)\mathcal{N}\mathcal{D}^{-1}/d^3}{3\epsilon_4}\,d^3E_{5z}^{\text{inc}}. \qquad \text{(A16)}$$

It is of some interest to check the results for the known case where the interior of the cell becomes homogeneous, i.e., there is no nucleus. Letting $\epsilon_2 = 1$ and $\epsilon_4 \equiv \epsilon$ in the final formulas (A7)–(A16) gives

$$\begin{aligned}\mathcal{D} &= -9[(1+2\epsilon)(2+\epsilon)-2(1-\epsilon)^2c^3/d^3], && \text{(A17)}\\ \mathcal{N} &= 9(d^3-c^3)(1+2\epsilon)(1-\epsilon), && \text{(A18)}\\ A_1 &= A_2' = A_3' = -\frac{9\epsilon}{(1+2\epsilon)(2+\epsilon)-2(1-\epsilon)^2c^3/d^3}\,E_{5z}^{\text{inc}}, && \text{(A19)}\\ A_2'' &= A_3'' = 0, && \text{(A20)}\\ A_4' &= -\frac{3(1+2\epsilon)}{(1+2\epsilon)(2+\epsilon)-2(1-\epsilon)^2c^3/d^3}\,E_{5z}^{\text{inc}}, && \text{(A21)}\\ A_4'' &= c^3\,\frac{3(1-\epsilon)}{(1+2\epsilon)(2+\epsilon)-2(1-\epsilon)^2c^3/d^3}\,E_{5z}^{\text{inc}}, && \text{(A22)}\\ A_5 &= -(d^3-c^3)\,\frac{(1+2\epsilon)(1-\epsilon)}{(1+2\epsilon)(2+\epsilon)-2(1-\epsilon)^2c^3/d^3}\,E_{5z}^{\text{inc}}, && \text{(A23)}\end{aligned}$$

in agreement with the corresponding formulas for the cell without nucleus [12].

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