AMSC/CMSC 466: HW #2
Due: Tuesday 2/9/16 (in class)

Please submit the solution to at least one problem in LaTeX.

1. The Maclaurin series for $(1+x)^n$ is also known as the binomial series. It states that

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots, \qquad (x^2 < 1).$$

   Derive this series. Then give its particular form in summation notation for $n = \frac{1}{2}$. Use this expression to compute $\sqrt{1.0001}$ correct to 15 decimal places.

2. Verify the following Taylor series and prove that it converges on the interval $-e < x \leq e$.

$$\ln(e+x) = 1 + \frac{x}{e} - \frac{x^2}{2e^2} + \frac{x^3}{3e^3} - \cdots = 1 + \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} \left(\frac{x}{e}\right)^k.$$

3. A function f is defined by the series

$$f(x) = \sum_{k=1}^{\infty} (-1)^k \left(\frac{x^k}{k^4}\right).$$

   Determine the minimum number of terms needed to compute $f(1)$ with error less than 10^{-8}.

4. What is the second term in the Taylor series of $(4x-1)^{\frac{1}{4}}$ about 4.25?

5. Expand the error function

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt,$$

   in a series by using the exponential series and integrating.