AMSC/CMSC 466: HW #3
Due: Tuesday 2/16/16 (in class)

Please submit the solution to at least one problem in LaTeX.

1. Use the bisection method to compute a positive root of $x^2 - 4x\sin x + (2\sin x)^2 - 1 = 0$ accurate to two significant figures.

2. Let the bisection method be applied to a continuous function, resulting in intervals $[a_0, b_0], [a_1, b_1], \ldots$. Let $r = \lim_{n\to\infty} a_n$. Are these statement true or false? Justify.

   (a) $a_0 \le a_1 \le a_2 \le \cdots$

   (b) $|r - 2^{-1}(a_n + b_n)| \le 2^{-n}(b_0 - a_0), \quad n \ge 0$

   (c) $|r - 2^{-1}(a_{n+1} + b_{n+1})| \le |r - 2^{-1}(a_n + b_n)|, \quad n \ge 0$

3. Perform four iterations of Newton's method for the polynomial $p(x) = 4x^3 - 2x^2 + 3$ starting with $x_0 = -1$.

4. What is the purpose of the following iteration formula?

$$x_{n+1} = 2x_n - x_n^2 y.$$

   Identify it as the Newton iteration for a certain function.

5. Halley's method for solving the equation $f(x) = 0$ uses the iteration formula

$$x_{n+1} = x_n - \frac{f_n f_n'}{(f_n')^2 - (f_n f_n'')/2}$$

   where $f_n = f(x_n)$ and so on. Show that this formula results when Newton's iteration is applied to the function $f/\sqrt{f'}$.

6. Show that the formula for the secant method can be written in the form

$$x_{n+1} = \frac{f(x_n)x_{n-1} - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}.$$

   Explain why, in practice, this formula is inferior to

$$x_{n+1} = x_n - f(x_n)\left[\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}\right], \quad n \ge 1.$$