AMSC/CMSC 466: HW #4
Due: Tuesday 2/23/16 (in class)

Please submit the solution to at least one problem in LaTeX.

1. Find the Lagrange and Newton forms of the interpolating polynomials for the following sets of data. Write both polynomials in the form $a+bx+cx^2$ in order to verify that they are identical.

   (a)

x	-2	0	1
$f(x)$	0	1	-1

   (b)

x	$-\sqrt{3/5}$	0	$\sqrt{3/5}$
$f(x)$	$f(-\sqrt{3/5})$	$f(0)$	$f(\sqrt{3/5})$

2. Establish an iteration method for solving $f(x) = 0$ as follows. Let $g_n(x)$ be the interpolating quadratic polynomial for the following data

x_{n-2}	x_{n-1}	x_n
$f(x_{n-2})$	$f(x_{n-1})$	$f(x_n)$

   Let x_{n+1} be the zero of $g_n(x)$ that is closest to x_n. What is x_{n+1}?

3. The equation $x - 9^{-x} = 0$ has a solution in $[0, 1]$. Find the interpolation polynomial on $x_0 = 0, x_1 = 0.5, x_2 = 1$ for the function on the left side of the equation. By setting the interpolation polynomial equal to zero and solving the equation, find an approximate solution to the equation.

4. Prove that if f is a polynomial of degree k, then for $n > k$, $f[x_0, x_1, \ldots, x_n] = 0$. (Hint: f is a polynomial).

5. Let $f(x) = 1/x$ and prove that

$$f[x_0, \ldots, x_n] = (-1)^n \prod_{i=0}^{n} x_i^{-1}.$$