

AMSC/CMSC 466: HW #5
Due: Tuesday 3/1/16 (in class)

Please submit the solution to at least one problem in LaTeX.

1. Show that the Chebyshev polynomials satisfy the differential equation

$$(1 - x^2)y'' - xy' + n^2y = 0, \quad n = 0, 1, 2, \dots$$

2. Compute the following values:

(a) $T_{2n+1}(0)$

(b) $T_{2n}(0)$

(c) $T_n(-1)$

3. Use the zeros of the Chebyshev polynomial $T_3(x)$ to construct an interpolating polynomial of degree two for the following functions on the interval $[-1, 1]$:

(a) $f(x) = e^x$

(b) $f(x) = \ln(x + 2)$

4. Find a bound for the interpolation errors in the previous problem.

5. Use the zeros of the Chebyshev polynomial $T_3(x)$ and transformations of the given interval to construct an interpolating polynomial of degree two for the following functions

(a) $f(x) = e^{-2x}$ on $[0, 2]$

(b) $f(x) = (x + 1) \ln x$ on $[1, 3]$