

AMSC/CMSC 466: HW #6
Due: Tuesday 3/22/16 (in class)

Please submit the solution to at least one problem in LaTeX.

1. Find a quartic polynomial (written in Newton's form) that takes these values: $p(0) = 2$, $p(1) = -4$, $p(2) = 44$, $p'(0) = -9$, and $p'(1) = 4$.
2. What condition will have to be placed on the nodes x_0 and x_1 if the interpolation problem

$$p(x_i) = c_{i0}, \quad p''(x_i) = c_{i2}, \quad i = 0, 1$$

is to be solvable by a cubic polynomial (for arbitrary c_{ij})?

3. Determine all the values of a, b, c, d, e for which the following function is a cubic spline

$$f(x) = \begin{cases} a(x-2)^2 + b(x-1)^3, & x \in (-\infty, 1], \\ c(x-2)^2, & x \in [1, 3], \\ d(x-2)^2 + e(x-3)^3, & x \in [3, \infty). \end{cases}$$

Next, determine the values of the parameters so that the cubic spline interpolates this table

x	0	1	4
y	26	7	25

4. Using the development of the cubic splines as a model, derive the appropriate equations and algorithms to provide a quadratic spline interpolant to data (t_i, y_i) for $0 \leq i \leq n$, where $t_0 < t_1 < \dots < t_n$. If Q is the spline interpolant, then the numbers $z_i = Q'(t_i)$ are well defined. Find the equations governing z_0, z_1, \dots, z_n . You should discover that one of the z points can be arbitrary, say $z_0 = 0$.
5. Determine the values of a, b, c so that this is a cubic spline having knots 0,1,2:

$$f(x) = \begin{cases} 3 + x - 9x^2, & x \in [0, 1], \\ a + b(x-1) + c(x-1)^2 + d(x-1)^3, & x \in [1, 2]. \end{cases}$$

Next, determine d so that $\int_0^2 [f''(x)]^2 dx$ is a minimum. Finally, find the value of d that makes f a natural cubic spline and explain why this value is different from the one previously determined.

6. Find a natural cubic spline function whose knots are $-1, 0, 1$ and that takes these values

x	-1	0	1
y	13	7	9