AMSC/CMSC 466: HW #7
Due: Thursday 3/31/16 (in class)

Please submit the solution to at least one problem in LaTeX.

1. Use the Gram-Schmidt process to construct the first three orthonormal polynomials for the following intervals (all with weight $w(x) = 1$): (a) $[0, 1]$. (b) $[0, 2]$. (c) $[-1, 3]$.

2. Find the linear least squares polynomial approximation to $f(x)$ in the indicated interval if

   (a) $f(x) = x^3 + 3x + 2$ on $[0, 1]$

   (b) $f(x) = \frac{1}{x}$ on $[-1, 3]$

   (c) $f(x) = \frac{1}{2}\cos x + \frac{1}{3}\sin 2x$ on $[-1, 1]$

3. Find the least squares polynomial approximations of degree two to the functions and intervals in the previous problem.

4. Find the first three orthonormal polynomial for the following weight functions $w(x)$ on the indicated intervals $[a, b]$:

   (a) $w(x) = \ln(x), \quad 0 \le x \le 1.$

   (b) $w(x) = x, \quad 0 \le x \le 1.$

   (c) $w(x) = \sqrt{1 - x^2}, \quad -1 \le x \le 1.$