AMSC/CMSC 466: HW #8
Due: Tuesday 4/12/16 (in class)

Please submit the solution to at least one problem in LaTeX.

1. Verify that the following two formulas actually approximate the third derivative. Find their error terms. Which formula is more accurate?

$$f'''(x) \approx \frac{1}{h^3}[f(x+3h) - 3f(x+2h) + 3f(x+h) - f(x)]$$

$$f'''(x) \approx \frac{1}{2h^3}[f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)]$$

2. Using Taylor series expansions, derive the error term for the formula

$$f''(x) \approx \frac{1}{h^2}[f(x) - 2f(x+h) + f(x+2h)].$$

3. Establish the most accurate formula of the form

$$f''_n \approx \frac{1}{h^2}[Af_{n+3} + Bf_{n+2} + Cf_{n+1} + Df_n]$$

Here $f_{n+i} = f(x_n + ih)$.

4. Derive the approximation

$$f'(x_n) \approx \frac{3f(x_n) - 4f(x_{n-1}) + f(x_{n-2})}{3x_n - 4x_{n-1} + x_{n-2}},$$

and show that the error term is $O(h^2)$ as $h \to 0$. Here $x_{n+i} = x_n + ih$.

5. Let L be an exact quantity that is approximated by $D(h)$ such that

$$L = D(h) + a_1 h + a_3 h^3 + a_5 h^5 + \dots$$

Use Richardson's extrapolation to obtain a third-order approximation of L. Use again Richardson's extrapolation to obtain a fifth-order approximation of L.