AMSC/CMSC 466: HW #9
Due: Tuesday 4/26/16 (in class)

1. Use the Lagrange interpolation polynomial to derive the formula of the form

$$\int_0^1 f(x)dx \approx Af(1/3) + Bf(2/3)$$

Transform the preceding formula to one for integration over $[a, b]$. Apply this result to evaluate $\int_0^\pi sin(x)$. Compare with the exact value of the integral.

2. Find the formula

$$\int_0^1 f(x)dx \approx A_0 f(0) + A_1 f(1)$$

that is exact for all functions of the form $f(x) = ae^x + b\cos(\pi x/2)$.

3. Use the Lagrange interpolation polynomial to derive the formula of the form

$$\int_0^1 f(x)dx \approx Af(0) + Bf(1/2) + Cf(1)$$

Transform the preceding formula to one for integration over $[a, b]$.

4. Derive a formula for approximating $\int_1^3 f(x)dx$, in terms of $f(0), f(1), f(4)$. It should be exact for all f in Π_2.

5. Derive the Newton-Cotes formula for $\int_0^1 f(x)dx$, based on the Lagrange interpolation polynomial at the nodes $-2, -1$ and 0. Apply this result to evaluate the integral when $f(x) = \sin \pi x$.

6. For what value of α is this formula exact on Π_3?

$$\int_0^2 f(x)dx \approx f(\alpha) + f(2 - \alpha).$$

7. Read Chapter 6 "Quadrature" in Cleve Moler's book and solve problems 6.3, 6.6. `http://www.mathworks.com/moler/quad.pdf`.