

The topics for the final.

In addition to the topic for midterm 1 and midterm 2 the following topic will be tested.

- (1) Biased and unbiased estimators
- (2) Minimum Variance Estimators
- (3) Method of moments
- (4) Method of maximum likelihood
- (5) Large sample confidence interval for population mean: $\bar{X} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$
- (6) Sample size needed to achieve a given confidence interval width $n = \left(\frac{2z_{\alpha/2}s}{w} \right)^2$.
- (7) Large sample upper and lower confidence bounds for population mean:

$$\mu < \bar{X} + z_{\alpha} \frac{s}{\sqrt{n}} \text{ and } \mu > \bar{X} - z_{\alpha} \frac{s}{\sqrt{n}}$$

- (8) Score confidence interval for proportion

$$\frac{\hat{p} + z_{\alpha/2}^2/2n}{1 + z_{\alpha/2}^2/n} \pm z_{\alpha/2} \frac{\sqrt{\hat{p}(1-\hat{p})/n + z_{\alpha/2}^2/4n^2}}{1 + z_{\alpha/2}^2/n}$$

where $\hat{p} = \frac{X}{n}$.

- (9) Score upper and lower confidence bounds for proportion

$$p < \frac{\hat{p} + z_{\alpha}^2/2n}{1 + z_{\alpha}^2/n} + z_{\alpha} \frac{\sqrt{\hat{p}(1-\hat{p})/n + z_{\alpha}^2/4n^2}}{1 + z_{\alpha}^2/n} \text{ and } p > \frac{\hat{p} + z_{\alpha/2}^2/2n}{1 + z_{\alpha/2}^2/n} - z_{\alpha} \frac{\sqrt{\hat{p}(1-\hat{p})/n + z_{\alpha}^2/4n^2}}{1 + z_{\alpha}^2/n}.$$

- (10) Confidence interval for mean in normal population $\bar{X} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$
- (11) Upper and lower confidence bounds for mean in normal population

$$\mu < \bar{X} + t_{\alpha, n-1} \frac{s}{\sqrt{n}} \text{ and } \mu > \bar{X} - t_{\alpha, n-1} \frac{s}{\sqrt{n}}$$

- (12) Prediction interval for normal population $\bar{X} \pm t_{\alpha/2, n-1} s \sqrt{1 + \frac{1}{n}}$

- (13) Type I and type II errors

- (14) Large sample hypothesis testing. Rejection regions:

- $\mu = \mu_0$ vs $\mu > \mu_0$: $\frac{\bar{X} - \mu_0}{s/\sqrt{n}} > z_{\alpha}$.
- $\mu = \mu_0$ vs $\mu < \mu_0$: $\frac{\bar{X} - \mu_0}{s/\sqrt{n}} < -z_{\alpha}$.
- $\mu = \mu_0$ vs $\mu \neq \mu_0$: $\left| \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \right| > z_{\alpha/2}$.

- (15) Large sample hypothesis testing. Type II error:

- $\mu = \mu_0$ vs $\mu > \mu_0$: $\Phi \left(z_{\alpha} + \frac{\mu_0 - \mu'}{s/\sqrt{n}} \right)$
- $\mu = \mu_0$ vs $\mu < \mu_0$: $1 - \Phi \left(-z_{\alpha} + \frac{\mu_0 - \mu'}{s/\sqrt{n}} \right)$
- $\mu = \mu_0$ vs $\mu \neq \mu_0$: $\Phi \left(z_{\alpha/2} + \frac{\mu_0 - \mu'}{s/\sqrt{n}} \right) - \Phi \left(-z_{\alpha/2} + \frac{\mu_0 - \mu'}{s/\sqrt{n}} \right)$

- (16) Hypothesis testing for normal population. Rejection regions:

- $\mu = \mu_0$ vs $\mu > \mu_0$: $\frac{\bar{X} - \mu_0}{s/\sqrt{n}} > t_{\alpha, n-1}$.
- $\mu = \mu_0$ vs $\mu < \mu_0$: $\frac{\bar{X} - \mu_0}{s/\sqrt{n}} < -t_{\alpha, n-1}$.
- $\mu = \mu_0$ vs $\mu \neq \mu_0$: $\left| \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \right| > t_{\alpha/2, n-1}$.

- (17) P-values