The topics for midterm 2.

(1) Binomial distribution $b(k;n,p)=\left(\begin{array}{c} n \\ k \end{array}\right)p^k(1-p)^{n-k}$. $EX=np$, $VX=np(1-p)$.

(2) Negative binomial distribution $nb(k;r,p)=\left(\begin{array}{c} r+k-1 \\ r-1 \end{array}\right)p^r(1-p)^k$.

$$EX=\frac{r(1-p)}{p},\ VX=\frac{r(1-p)}{p^2}.$$

(3) Hypergeometric distribution $h(k;n,M,N)=\frac{\left(\begin{array}{c} M \\ k \end{array}\right)\left(\begin{array}{c} N-M \\ n-k \end{array}\right)}{\left(\begin{array}{c} N \\ n \end{array}\right)}$.

$$EX=n\frac{M}{N},\ VX=n\frac{M}{N}\left(1-\frac{M}{N}\right)\frac{N-n}{N-1}.$$

(4) Poisson distribution $p(k,\lambda)=e^{-\lambda}\frac{\lambda^k}{k!}$. $EX=\lambda$, $VX=\lambda$.

(5) Poisson process $N(a,b)\sim$Pois$(\lambda(b-a))$.

(6) Desnisty and cumulative distribution function of continuous distributions

$$F_X(x)=P(X\leq x),\quad P(a\leq X\leq b)=\int_a^b f_X(x)dx,\quad f(x)=F'(x).$$

(7) Median and other percentiles $F_X(m)=0.5,\quad F_X(x_k)=\frac{k}{100}$.

(8) Uniform distribution $f(x)=\frac{1}{b-a}$, $EX=\frac{b+a}{2}$, $VX=\frac{(b-a)^2}{2}$.

(9) Expected value of continuous random variable

$$E(X)=\int_{-\infty}^{\infty}xf_X(x)dx,\quad E(h(X))=\int_{-\infty}^{\infty}h(x)f_X(x)dx.$$

(10) Variance $V(X)=E(X^2)-(EX)^2$.

(11) Normal distribution $f(x;\mu,\sigma^2)=\frac{1}{\sigma\sqrt{2\pi}}\exp-\frac{(x-\mu)^2}{2\sigma^2}$

$$X=\mu+\sigma Z \text{ where } Z\sim N(0,1),\quad P(a\leq X\leq b)=\Phi\left(\frac{b-\mu}{\sigma}\right)-\Phi\left(\frac{a-\mu}{\sigma}\right).$$

(12) Binomial approximation for normal distribution $\mathrm{Bin}(n,p)\approx N(np,np(1-p))$.

(13) Exponential distribution $f(x;\lambda)=\lambda e^{-\lambda x}$. $F(x;\lambda)=1-e^{-\lambda x}$. $EX=\frac{1}{\lambda}$, $VX=\frac{1}{\lambda^2}$.

(14) Gamma distribution $f(x;\alpha,\beta)=\frac{1}{\beta^\alpha\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta}$. $EX=\alpha\beta$, $VX=\alpha\beta^2$.

(15) Joint distribution

$$P((X,Y)\in A)=\sum_{(x,y)\in A} p(x,y) \text{ discrete distributions,}$$

$$P((X,Y)\in A)=\iint_A p(x,y)dxdy \text{ continuous distributions.}$$

(16) Marginals

$$p_X(x)=\sum_y p(x,y),\quad p_Y(y)=\sum_x p(x,y), \text{ discrete distributions,}$$

$$p_X(x)=\int p(x,y)dy,\quad p_Y(y)=\int p(x,y)dx, \text{ continuous distributions.}$$

(17) Conditional distributions $p_{X|Y}(x|y)=\frac{p(x,y)}{p_Y(y)}$.

(18) Two dimensional uniform distribution $f(x,y)=\dfrac{1}{Area(D)}$ if $(x,y)\in D$. $P((X,y)\in A)=\dfrac{Area(A)}{Area(D)}$.

(19) Independent Random Variables $p_{X,Y}(x,y)=p_X(x)p_Y(y)$.

(20) Sum of independent random variables $Z=X+Y$.

$$f_Z(z)=\sum f_X(x)f_Y(z-x) \text{ discrete distributions,}$$

$$f_Z(z)=\int f_X(x)f_Y(z-x)dx \text{ continuous distributions,}$$

(21) Expectation

$$E(h(X,Y))=\sum\sum h(x,y)p(x,y) \text{ discrete distributions,}$$

$$E(h(X,Y))=\iint h(x,y)p(x,y)dxdy \text{ continuous distributions,}$$

(22) Properties of expectation
- (a) $E(X+Y)=E(X)+E(Y)$,
- (b) $E(aX+b)=aE(X)+b$,
- (c) If X and Y are independent $E(XY)=E(X)E(Y)$.

(23) Covariance $\text{Cov}(X,Y)=E(XY)-(EX)(EY)$.

(24) Properies of covariance
- (a) $\text{Cov}(X,X)=V(X)$,
- (b)$\text{Cov}(aX+b,cY+d)=ac\text{Cov}(X,Y)$,
- (c) $\text{Cov}(\sum_i a_jX_i,\sum_j b_jY_j)=\sum_{ij} a_ib_j\text{Cov}(X_i,Y_j)$,
- (d) If X and Y are independent $\text{Cov}(X,Y)=0$.

(25) Properites of variance

$$V(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} V(X_i) + 2\sum_{i<j} \mathrm{Cov}(X_i, X_j);$$

If X_i are independent then $V(X_1 + X_2 + \dots X_n) = V(X_1) + V(X_2) + \dots V(X_n)$.

(26) Central Limit Theorem X_i are independent identically distributed

If $S = X_1 + X_2 + \dots X_n$ then $S \approx N(E(S), V(S))$.

(27) Multivariate Gaussian distribution. If $(X_1, X_2, \dots X_n)$ is multivariate normal then $\sum_{j=1}^{n} a_j X_j$ is normal.