

**STAT400. Final.**

**NAME:**

**STUDENT NUMBER:**

(1) A set menu in a teahouse consists of a cake and a cup of tea. There are 10 different cakes and 8 different kinds of tea.

(a) How many different set menus are available?

(b) If two customers order set menus independently what is the probability that they order either the same cake or the same tea?

(c) If six people order independently what is the probability that all order different sets? (Two sets are considered different if either cakes or teas are different)

(2) 1 % of all athletes use a certain drug. If a person uses this drug it can be detected by a drug test with probability 0.95. If the person does not use the drug they can be falsely accused with probability 0.05.

(a) If a random athlete is chosen what is the probability that the test result is positive.

(b) If the result is positive what is the (conditional) probability that the athlete is using the drug?

(3) In proof testing of circuit boards the probability that any particular diode will fail is 0.01. Suppose a circuit board contains 90 diodes.

(a) How many diodes would you expect to fail and what is the standard deviation of the number that are expected to fail?

(b) What is the (approximate) probability that at least two diodes will fail on a randomly selected board?

(c) If ten boards are shipped to a particular customer how likely (approximately) is what at least four of them will have no failed diodes?

(4) Let  $X$  be a random variable with density  $p(x) = x^3/4$  if  $0 \leq x \leq 2$  and  $p(x) = 0$  elsewhere.

(a) Compute  $E(X)$

(b) Compute  $V(X)$

(c) Let  $Y = X^2$ . What is the probability that  $Y > 1$ ?

(d) Find 75th percentile of  $X$ .

(5) The distance  $X$  of a random taxi ride has uniform distribution on the segment  $[2,10]$  miles.

(a) Compute  $EX$ .

(b) Compute  $VX$ .

A cab driver takes 48 customers a week. Let  $D$  be the total distance traveled by all his customers.

(c) Compute  $ED$ .

- (d) Compute  $VD$ .
- (e) Compute approximately the probability that  $280 < D < 300$ .
- (6) Let  $X$  have the distribution with density  $p(x) = (s + 1)x^s$ , if  $0 \leq x \leq 1$  and 0 otherwise. Here  $s > -1$  is an unknown parameter. Given the following sample from this distribution  
0.98 0.54 0.67 0.44 0.92  
estimate the value of  $s$ .