Variance, Covariance and Central Limit Theorem.

1. *A doctor prepared bills for 10 patients. Her absent minded secretary put 10 bills into 10 addressed envelops randomly. Let $X_i = 1$ if the i-th bill was sent ot a correct address.*

(a) Compute EX_1, VX_1, $Cov(X_1, X_2)$,

(b) Let X be number of bills sent to the correct address. Compute $E(X)$ and $V(X)$.

2. *Compute EX and VX if*

(a) $X \sim Bin(n, p)$, (b) $X \sim Hyp(N, M, n)$, (c) $X \sim NBin(n, p)$.

3. *Suppose 50 % of all drivers have American cars, 40 % have Japanese cars and 10 % have European cars. Consider 15 consecutive cars crossing certain intersection. Let A be the number of American cars among those 15 cars and J be the number of Japanese cars. Compute $Cov(A, J)$, $Corr(A, J)$.*

4. *Fruitland's economy consists of two companies. Each issue shares costing 10 oranges/share. During the next year Maracuja's shares will grow 2 oranges with probability 1/2 or drop 1 orange with probability 1/2. PassionFriuit's shares will grow 3 oranges with probability 1/2 or drop 2 oranges with probability 1/2 independent of Maracuja. You have 360 oranges (an orange is the name of Fruitland's currency so you can not eat it). How much should you invest into each company to minimize you risk?*

5. *John takes bus to work 300 days a year. Suppose that bus waiting times are independent of one another and each is uniformly distributed between 0 and 10 min. Let T be total waiting time for a particular year.*

(a) Compute $E(T)$ and $V(T)$.

(b) Compute approximately the probability that T is more than 26 hours; that it is between 24 and 25 hours.

6. *A certain lamp uses lightbulbs whose lifetime have exponential distribution with mean 100 hours. A storage room has a supply of 100 bulbs. Find the probability that this supply would not suffice for 9500 hours of work.*

7. *Let $X \sim Pois(100)$. Compute approximately $P(100 \leq X \leq 105)$.*

8. *Let (X_1, X_2) be a normal vector with mean $(1, 2)$ and covariance matrix $\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$. Compute $P(X_2 > X_1)$.*