

Point estimation.

1. Let X_1, X_2, X_3, X_4, X_5 be independent random variables each having uniform distribution on the interval $[0, \theta]$. Consider the following estimators on the unknown parameter θ :

$$\hat{\theta}_1 = 2\bar{X}; \hat{\theta}_2 = X_{(5)}; \hat{\theta}_3 = \frac{6}{5}X_{(5)}; \hat{\theta}_4 = 2X_{(3)}.$$

Which of the above estimators are unbiased? Which has the smallest variance?

2. Estimate parameters of the following distributions using the method of moments:

(a) $N(\mu, \sigma^2)$.

(b) $\Gamma(\alpha, \beta)$;

(c) $Uni(a, b)$.

(d) $Bernoulli(p)$.

3. Find the MLE for the following distributions:

(a) $N(\mu, \sigma^2)$.

(b) $Exp(\lambda)$.

(c) $Uni(a, b)$.

(d) $Bernoulli(p)$.