Hypothesis testing.

1. *In each of the cases below state appropriate null and alternative hypotheis:*

(a) The drug which is currently in use decreases the systolic blood pressure of patients with certain condition by 20 units. The doctor wants to decide if the new drug should be used instead.

(b) A doctor noticed that a number of her patients had stroke after taking certain diabetis medication. She asks a researcher to determine if taking the drug increases the probability of stroke.

(c) A ring in a certain museum is claim to belong to Richard the Lionheart. Since some doubts were made concerning rings authenticity carbon dating of the ring was performed.

(d) To detrmine if two SAT tests have same level of difficulty 100 students were asked to take both test and the difference in scores were recorded for each student.

(e) Producer claims that each serving of juce contains 10g of sugar.

In each case explain in words the meaing of type I and type II errors.

2. *Two different companies have applied to provide cable television service in a certain region. Let p denote the proportion of all potential subscribers who favor the first company over the second. Consider testing $H_0 : p = .5$ versus $H_a: p \geq .5$ based on a random sample of 25 individuals. Let X denote the number in the sample who favor the first company and x represent the observed value X. For each of the following rejection regions find type I error as well as type II error in case $p = 0.75$*

$$x \geq 15, \quad x \geq 16, \quad x \geq 17, \quad x \geq 18, \quad x \geq 19.$$

3. *A mixture of pulverized fuel ash and Portland cement to be used for grouting should have a compressive strength of more than 1300 KN/m^2. The mixture will not be used unless experimental evidence indicates conclusively that the strength specification has been met. Suppose compressive strength for specimens of this mixture is normally distributed with $\sigma = 60$. Let μ denote the true average compressive strength. 25 observations are made.*

(a) State H_0 and H_a;
(b) Construct rejection region with confidence level $\alpha = 0.05$.
(c) Compute the probability of type II error if $\mu = 1320$.

4. *The drying time of a certain type paint under specified test conditions is known to be normally distributed with mean value 75 min and a standard deviation, $\sigma = 9$ min. Chemists have proposed a new additive designed to decrease average drying time. It is believed that drying times with this additive will remain normally distributed with $\sigma = 9$. Because of the expense associated with the additive, evidence should strongly suggest an improvement in average drying time before such a conclusion is adopted. Experimental data is to consist of dying times from $n = 25$ test specimens and $\overline{x} = 72.3$. Perform test for this problem using $\alpha = .01$.*

5. *The drying time of a certain type paint under specified test conditions is known to be normally distributed with mean value 75 min. Chemists have proposed a new additive designed to decrease average drying time. It is believed that drying times with this additive will remain normally distributed. Because of the expense associated with the additive, evidence should strongly suggest an improvement in average drying time before such a conclusion is*

adopted. Experimental data is to consist of dying times from $n = 100$ test specimens. From this sample $\bar{x} = 72.3$ min and $s = 9$ min.

Perform test for this problem using $\alpha = .01$.

6. *Producer claims that his packages contain on average 100 g of walnuts. 49 poackages were examined resulting in $\bar{x} = 92$ and $s = 20$*

(a) Test hypothesis $H_0 = \{\mu = 100\}$ versus $H_a = \{\mu \neq 100\}$ at cofidence level $\alpha = 0.05$.
(b) Estimate the power of the test if $\mu = 95$.
(c) Compute 5% confidence interval for the average walnut contence.

Solution. (a) We test $H_0 = \{\mu = 100\}$ versus $H_a = \{\mu \neq 100\}$. Test statistics is $z = \frac{\bar{x}-100}{s/\sqrt{n}}$. Rejection region is $|z| \geq z_{0.025} = 1.96$. In our case $z = \frac{92-100}{20/7} = -2.8$. Since this number falls into rejection region we have sufficient evidence to claim that the label is inaccurate.

(b) The formula for type II error for two sided alternative is

$$\Phi\left(z_{\alpha/2} + \frac{\mu_0 - \mu'}{s/\sqrt{n}}\right) - \Phi\left(-z_{\alpha/2} + \frac{\mu_0 - \mu'}{s/\sqrt{n}}\right) = \Phi\left(1.96 + \frac{100-95}{20/7}\right) - \Phi\left(-1.96 + \frac{100-95}{20/7}\right)$$

$$= \Phi(3.71) - \Phi(-0.21) \approx 1 - 0.42 = 0.58.$$

Accordingly the power is about $1 - 0.58 = 0.42$.

(c) The confidence interval takes form

$$\bar{x} \pm z_{0.025}\frac{s}{\sqrt{n}} = 92 \pm \frac{1.96 * 20}{3} = 92 \pm 5.6 = [86.4, 97.6].$$

Note that the confidence interval does not contain 100 which is equivalent to the fact that we rejected H_0 in part (a).

7. *The true average voltage drop from collector to emitter of insulated gate bipolar transistors of a certain type is supposed to be at most 2.5 volts. Assume that the voltage drop is normally distributed. An investigator selects a sample of n=10 such transistors and uses the resulting voltages as a basis for testing H_0: $\mu = 2.5$ versus H_a: $\mu > 2.5$ using a t test with significance level $\alpha = .05$. What are test conclusins if $\bar{x} = 2.6$ and $s = 0.2$?*

Solution. We test H_0: $\mu = 2.5$ versus H_a: $\mu > 2.5$ The test statistic is $t = \frac{\bar{x}-2.5}{s/\sqrt{n}}$. Since the data is normal the rejection region is $t > t_{0.05,9} = 1.883$. In our case $t = \frac{2.6-2.5}{0.2/\sqrt{10}} = 1.581$. Since it does not fall in the rejection region we do not reject H_0. That is there is not enough evidence that the average voltage drop is above the required norm.

8. *A county needs to levy taxes to repair a section of the road. A survey of 100 roads nearby resulted in the following cost per mile $\bar{x} = 888K$, $s = 215K$. Test the hypothesis $H_0 = \{\mu = 1000\}$ against the alternative $H_a = \{\mu \neq 1000\}$ at the significance level $\alpha = 0.01$. How confident are in the results of the test?*

Solution. We test $H_0 = \{\mu = 1000\}$ versus $H_a = \{\mu \neq 1000\}$. Test statistics has form $z = \frac{\bar{x}-1000}{s/\sqrt{100}}$. Rejection region is $|z| > z_{0.005} = 2.58$. In our case $z = \frac{888-1000}{215/10} = -5.21$. Since it falls in the rejection region we have sufficient evidence to the actual cost of the road is different (in fact, less than) 1000K/mile.

In fact using online calculator for Φ we have $P(|Z| > 5.21) = 2P(Z > 5.21) \approx 3.46 \times 10^{-13}$. Thus if H_0 was true we have less than 1 chance in a trillion to get a value more contradictory

to the null hypothesis than the present one. So we can be pretty confidence in the results of the test.

9. *Pairs of P-values and significance levels, α, are given. For each pair, state whether the observed P -value would lead to rejection of H_0 at the given significance level.*
   (a) P-value=.084, α = .05.
   (b) P-value=.003, α = .001.
   (c) P-value=.498, α = .05.
   (d) P-value=.084, α = .10.
   (e) P-value=.039, α = .01.
   (f) P-value=.218, α = .10.
   (g)P-value=.012, α = .05.

Solution. (a) $0.084 > 0.05$, do not reject H_0.
   (b) $0.003 > 0.001$, do not reject H_0.
   (c) $0.498 > 0.05$, do not reject H_0.
   (d) $0.084 < 0.10$, reject H_0.
   (e) $0.039 > 0.01$ do not reject H_0.
   (f) $0.218 > 0.10$, do not reject H_0.
   (g) $0.012 < 0.05$, do reject H_0.

10. *For the following z test statistics find P-values for each type of alternative hypothesis and make conclusion at significance level α = .10, and α = .01*
   (a) z=1.42; (b) z=-.11 (c) z=2.10

Solution. The answer is contained in the following table. R means reject, NR–not reject. First answer refers to α = .10, the second to α = 0.01.

H_a	$\{\mu > \mu_0\}$	$\{\mu < \mu_0\}$	$\{\mu \neq \mu_0\}$
1.42	0.08, R, NR	0.92, NR, NR	0.16, NR, NR
-.11	0.54, NR, NR	0.46, NR, NR	0.92, NR, NR
2.10	0.02, R, NR	0.98, NR, NR	0.04 R, NR