

1. Let  $A$ ,  $B$  and  $C$  are three events such that  $P(A) = 0.45$ ,  $P(B) = 0.30$ ,  $P(C) = 0.35$ ,  $P(A \cup B) = 0.60$ ,  $P(A \cup C) = 0.60$ ,  $P(B \cup C) = 0.50$ ,  $P(A \cup B \cup C) = 0.70$ .
- Compute  $P(A \cap B)$ ,  $P(A \cap C)$ ,  $P(B \cap C)$ .
  - Compute  $P(A \cap B \cap C)$ .
  - Compute the probability that exactly one of  $A$ ,  $B$  and  $C$  happens.
2. You are dealt a 2-card hand from a well-shuffled deck of 52.
- What is the probability of the union of the event that both cards are aces with the event that both cards are red (hearts or diamonds)?
  - That is the probability that both cards have the same denomination?
  - What is the conditional probability that the second card is a picture card ( $J$ ,  $Q$ , or  $K$ ) given that at least one of the two cards is a picture card ?
3. In a certain city there are two food stores. 70% of the the population use Cheap FoodStore and 30% use Green FoodStore. Among the shoppers of Cheap FoodStore 70% have household income of less than 50000 per year while among the shoppers of Green FoodStore 50% have household income of less than 50000 per year.
- Find the probability that a randomly chosen citizen uses Green FoodStore and has household income 50000 or more per year.
  - What proportion of the citizens have household income 50000 or more per year.
  - Given that a person has a household income 50000 or more per year how likely he is to shop at Green FoodStore.
4. Let  $X$  have cumulative distribution function  $F_X(x) = x^2$  for  $0 \leq x \leq 1$  (and  $F_X(x) = 0$  for  $x < 0$  and  $= 1$  for  $x > 1$ ),
- Find the density of  $f_X(x)$ ;
  - find the probability density function  $f_V(v)$  of  $V = 3X + 1$ ;
  - Compute the median, 25th and 75th percentiles of  $X$ .
5. The continuous random variable  $Y$  has density  $f_Y(y) = \frac{2}{27}y(y + 1)$  for  $0 \leq y \leq 3$  (and  $f_Y(y) = 0$  otherwise).
- Find the cumulative distribution function of  $Y$ .
  - Find  $E(1/Y)$ .
  - If  $Y_1, Y_2 \dots Y_{10}$  are independent random variables with distribution  $Y$  find the distribution of  $M = \max(Y_1, Y_2 \dots Y_{10})$ .
6. 1000 independent random variables  $W_j$ ,  $j = 1, \dots, 1000$  have been simulated, according to an exponential distribution with parameter  $\lambda = 1$ .
- Let  $N$  be the number among the variables  $W_j$  which are larger than  $\ln(500)$ . What is the exact probability distribution of  $N$ ?
  - What is the approximate probability that  $N$  is 3 or less?
  - Answer questions (a), (b) if the number  $\ln(500)$  is replaced by  $\ln(5)$ .

7. Let  $(X, Y)$  have joint density  $f_{X,Y}(x, y) = 2xy + 8x^3y^3$  if  $0 \leq x \leq 1$  and  $0 \leq y \leq 1$  (and  $f_{X,Y}(x, y) = 0$  otherwise).
- Compute the marginal density of  $X$ .
  - Compute  $EX$  and  $VX$ .
  - Find the  $Cov(X, Y)$ .
8. Toll rates at a certain bridge is \$ 2 for 2 axis vehicles, \$ 4 for 3 axis vehicles and \$ 10 for vehicles with 4 or more axis. Suppose that 80% of cars passing the bridge have 2 axis, 10% have 3 axis and 10% have 4 or more axis. Let  $S$  be the toll collected from next 1 million cars passing the bridge.
- Compute  $ES$ .
  - Compute  $VS$ .
  - Compute approximately  $P(S > 3003000)$ .
9. Suppose that  $X$  and  $Y$  are independent random variables with  $E(X) = 1, \sigma_X^2 = 2, E(Y) = 4, \sigma_Y^2 = 3$ . Let  $U = 5X - 4Y$ .
- Find the mean and variance of  $U$ .
  - Compute  $Cov(X, U)$ .
  - If  $X$  and  $Y$  are each normally distributed find  $P(U \geq 1)$ .
10. Let  $X_1, X_2, \dots, X_5$  be a sample of some unknown distribution  $X$ .  
Let  $\hat{\mu} = \frac{X_1}{2} + \frac{X_2}{4} + \frac{X_3}{8} + \frac{X_4}{16} + \frac{X_5}{16}$  be an estimator of the population mean.
- Is  $\hat{\mu}$  unbiased or not?
  - Express  $V(\hat{\mu})$  in terms of  $V(X)$ .
  - Compare  $\hat{\mu}$  with the sample mean.
11. Suppose that  $W$  is a discrete random variable such that  $P(W = 0) = (1 - a)/4, P(W = 1) = (1 + 2a)/2$ , and  $P(W = 2) = (1 - 3a)/4$ , where  $-\frac{1}{2} < a < \frac{1}{3}$  is an unknown parameter.
- Compute  $EW$ .
- Let  $(2, 0, 1, 1, 1, 2)$  be the sample from the distribution  $W$ .
- Find the method of moment estimator for  $a$ .
  - Find the maximum likelihood estimator of  $a$ .
12. Let  $X$  be a random variable with density  $f_X(x) = 1 - \theta + 2\theta x$  if  $0 \leq x \leq 1$  (and  $f_X(x) = 0$  otherwise) where  $0 < \theta < 1$  is an unknown parameter.
- Compute  $EX$  and  $VX$ .
- Let  $X_1, X_2 \dots X_n$  be a sample of this distribution.
- Find the method of moment estimator for  $\theta$ .
  - Give an estimate for the variance of the estimator from part (b).
13. After examining 50 bags of rice in a warehouse an inspector came with the following 95% confidence interval for the average weight of a bag  $9.9 \pm 0.2$  lbs.
- Compute 99% confidence interval for the average weight of a bag;

- (b) Compute 95% lower confidence bound for the average weight of a bag;
- (c) Estimate how many bags need to be examined so that the width of the 95% confidence interval is 0.1 lbs.

**14.** Researchers investigated the physiological changes that accompany laughter. Ninety (90) subjects (18 – 34 years old) watched film clips designed to evoke laughter. During the laughing period, the researchers measured the heart rate (beats per minute) of subject, with the following summary results:  $\bar{x} = 73.1$ ,  $s = 3$ . It is well known that the mean resting heart rate of adults is 71 beats per minute.

(a) Test whether the true mean heart rate during laughter exceeds 71 beats per minute using  $\alpha = .05$ .

(b) Find the power of the test at  $\mu = 72$

(c) Estimate true mean heart rate during laughter in a way that conveys information about precision and reliability. (Calculate 95% CI).

**15.** A business journal investigation of the performance and timing of corporate acquisitions discovered that in a random sample of 2,863 firms, 848 announced one or more acquisitions during the year 2000. Let  $p$  be the proportion of firms which made one or more acquisitions during the year 2000.

(a) Give a point estimate for  $p$ ;

(b) Construct 90% confidence interval for  $p$ ;

(c) Construct 90% upper confidence bound for  $p$ .

**16.** Strategic placement of lobster traps is one of keys for a successful lobster fisherman. A study was conducted of the average distance separating traps by lobster fishermen. The trap-spacing measurements (in meters) for a sample of seven teams from Blue Sea fishing cooperative and came with the following data:  $\bar{x} = 81.00$  and  $s_1^2 = 125$ . Suppose that the trap-spacing has normal distribution.

(a) Test the hypothesis that the average distance between the traps is at least 90m at the significance level  $\alpha = 0.05$ .

(b) What is the problem with using the normal ( $z$ ) statistic to find a confidence interval for the average distance between the traps?

(c) Construct 95% confidence interval for the average distance between the traps.

(d) One team from Blue Sea cooperative was not working during the day of test. Given 95% prediction interval for the distance between the traps used by the missing team.