STAT400. Sample questions for midterm 1.

1. *Let A and B are sets such that $P(A) = 0.6$, $P(B) = 0.4$ and $P(AB) = 0.3$.*

(a) Compute $P(A \cup B)$.

(b) Compute $P(A|B)$.

(c) Compute $P((A \cup B)')$.

2. *Let A denote the event that a customer at a certain store has visa, B denote the event that she has a master card and C denote the event that she has a discovery card. Suppose that $P(A) = 0.70$, $P(B) = 0.63$, $P(C) = 0.33$, $P(AB) = 0.50$, $P(AC) = 0.25$, $P(BC) = 0.23$, $P(A \cup B \cup C) = 0.88$. Compute the probabilities that a customer*

(a) owns none of the cards;

(b) has all three types;

(c) have exactly one type.

3. *How many ways are there to distribute 6 different toys and 9 differnt candies between 3 children*

(a) without any restrictions;

(b) If the first child needs to get exactly two toys and 3 candies;

(c) If all children need to get exactly two toys and 3 candies.

4. *(a) How many ways are there to divide 52 cards between four people so that each person gets 13 cards.*

(b) Find the probability that players 1 and 3 have two aces each.

(c) Find the probability that player 1 has all his cards red (that is, they are either hearts or diamonds) and has exactly one ace.

5. *The first box contains 2 red and 3 blue balls and the second box has 3 red and 2 blue balls. A ball is chosen at random from the first box and put in the second box. Then a ball is chosen at random from the second box.*

(a) Find the probability that the first ball is blue;

(b) Find the probability that both balls are blue;

(c) Find the probability that the balls have the same color.

6. *In a certain city 70% of drivers are careful and 30% are agressive. Assume that a careful driver has 10% chance of getting a speed ticket independent of the past performance and an agressive driver has 30% chance of getting a speed ticket independent of the past performance.*

(a) Find the probability that a careful driver will get a ticket during the first year but not during the second year.

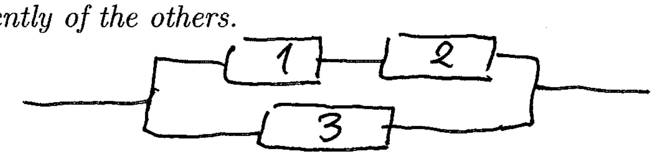
(b) Find the probability that an agressive driver will get a ticket during the first year but not during the second year.

(c) Given that a driver was ticket during the first year but not during the second year find the probability that he is agressive.

7. *There are two identical urns. The first urn contains 5 balls numbered 1 to 5. The first urn contains 10 balls numbered 1 to 10. One urn is chosen at random and then 3 balls are selected from that urn without replacement. Let A be the event that the first urn is chosen, B be the event that the second urn is chosen and C be the event that the maximum number of the balls chosen is 4. Compute*

(a) $P(C|A)$ *and* $P(C|B)$;
(b) $P(C)$;
(c) $P(A|C)$.

8. *Consider the chain below. Suppose that each element works with probability $2/3$ independently of the others.*



(a) *Find the probability that the chain works.*
(b) *Find the probability that the chain works given that the first element works.*
(c) *Find the probability that the first element works given that the chain works.*

9. *Let A, B and C be mutually independent and $P(A) = 1/2$, $P(B) = 1/3$, $P(C) = 1/4$. Let D be the event that exactly one of events A, B and C occurs.*

(a) *Compute* $P(D)$.
(b) *Compute* $P(A|D)$, $P(B|D)$, $P(C|D)$.
(c) *Compute* $P(A|(B \cup C))$.

10. *An urn contains 4 red and 6 blue balls. Balls are taken at random without replacement until both colors are present.*

(a) *Find the probability that two balls are enough.*
(b) *Find the probability that three balls are not enough.*
(c) *Find the probability that exactly three balls are needed.*

11. *An urn has 5 red, 5 green and 5 blue balls. 3 balls are chosen at random (without replacement). Let X be the number of different colors chosen.*

(a) *Compute the probability mass function of X.*
(b) *Compute the cumulative distribution function of X.*
(c) *Compute EX and VX.*

12. *The probability mass function of X is given in the following table*

X	0	1	2	3
p	0.1	0.2	0.3	0.4

Let $Y = (X-1)^2$.

(a) *Compute the probability mass function of Y.*
(b) *Compute the cumulative distribution function of Y.*
(c) *Compute EY and VY.*