Workshop on Dynamical Systems and Related Topics 2015

Friday – March 31.

9:00 – 9:30  Coffee and Registration
9:30 – 10:30  Yuri Bakhtin
10:45 – 11:45  Kening Lu
11:45 – 12:15  Coffee break
12:15 – 1:15  Zsolt Pajor-Gyulai
1:15 – 3:15  Lunch
3:15 – 4:15  Dmitry Jakobson (Department Colloquium)
4:15 – 5:15  Alex Grigo

Saturday – April 1.

9:00 – 9:30  Coffee
9:30 – 10:30  Anthony Quas
10:30 – 11:00  Coffee break
11:00 – 11:40  Parallel Session-1
   Changguang Dong (MTH3206)
   Farruh Shahidi (MTH0307)
11:45 – 12:25  Parallel Session-2
   Adam Zydney (MTH3206)
   Agnieszka Zelerowitz (MTH0307)
12:30 – 1:10  Parallel Session-3
   Yuki Takahashi (MTH3206)
   Wenyu Pan (MTH0307)
1:10 – 3:00  FREE Lunch (sandwiches)
3:00 – 4:00  Yakov Pesin
4:10 – 4:50  Parallel Session-4
   Pratima Hebbar (MTH3206)
   Alena Erchenko (MTH0307)
4:50 – 5:10  Coffee break
5:10 – 6:10  Victoria Sadovskaya
6:30 – 9:00  Banquet

Sunday – April 2.

9:00 – 10:00  Andrew Török
10:10 – 11:10  Tobias Hurth
11:10 – 11:40  Coffee break
11:40 – 12:40  Alexei Novikov
12:50 – 1:50  Ke Zhang
Titles and Abstracts.

**Yuri Bakhtin** *Ergodic theory of the Burgers equation with random forcing.*

The Burgers equation is a basic nonlinear evolution PDE of Hamilton–Jacobi type related to fluid dynamics and growth models. I will talk about the ergodic theory of randomly forced Burgers equation in noncompact setting. The basic objects are one-sided infinite minimizers of random action (in the inviscid case) and polymer measures on one-sided infinite trajectories (in the positive viscosity case). This is joint work with Eric Cator, Kostya Khanin, and Liying Li.

**Tobias Hurth** *Smoothness of invariant densities for switching systems on $\mathbb{T}^2$.*

In this talk, we describe a class of dynamical systems with random switching between smooth vector fields on the two-dimensional torus. These switching systems admit an absolutely continuous invariant measure, and the corresponding density is smooth. We present an argument based on integration by parts that yields smoothness of the density, and discuss in how far this approach can be used to study invariant densities for more general switching systems on finite-dimensional smooth manifolds. This talk is based on work with Yuri Bakhtin, Sean Lawley and Jonathan Mattingly.

**Dmitry Jakobson** *Quantum ergodicity for ray-splitting (branching) billiards.*

After giving an overview of Quantum Ergodicity results on compact Riemannian manifolds with ergodic geodesic flow (due to Shnirelman, Zelditch, Colin de Verdiere and others), we discuss joint work with Yury Safarov and Alex Strohmaier, which concerns the semiclassical limit of spectral theory on manifolds whose metrics have jump-like discontinuities. Such systems are quite different from manifolds with smooth Riemannian metrics because the semiclassical limit does not relate to a classical flow but rather to branching (ray-splitting) billiard dynamics. In order to describe this system we introduce a dynamical system on the space of functions on phase space. We prove a quantum ergodicity theorem for discontinuous systems. In order to do this we introduce a new notion of ergodicity for the ray-splitting dynamics. If time permits, we outline an example (provided by Y. Colin de Verdiere) of a system where the ergodicity assumption holds for the discontinuous system. We end with a list of open problems.

**Kening Lu** *Wong-Zakai approximations and dynamics of stochastic differential equations.*

We study the Wong-Zakai approximations given by a stationary process via the Wiener shift and their associated path-wise dynamics of the stochastic differential equation driven by a $n$-dimensional Brownian motion. We prove that the solutions of Wong-Zakai approximations converge in the mean square to the solutions of the Stratonovich stochastic differential equation. We also show that for a simple multiplicative noise, the center-manifold of the Wong-Zakai approximations converges to the center-manifold of the Stratonovich stochastic differential equation. Assuming the equation with only drift term has a homoclinic orbit to a saddle fixed point, we prove that if the diffusion term is not completely tangent to the homoclinic orbit, then for almost all sample paths of the Brownian motion, the forced equation admits a topological horseshoe of infinitely many branches, thus is chaotic.

**Alexei Novikov** *A survival guide for feeble fish.*

This is a joint work with D.Burago and S.Ivanov. As avid anglers we were always interested in the survival chances of fish in turbulent oceans. In this talk I will address this question mathematically, and discuss some of its consequences. I will show that a fish with bounded aquatic locomotion speed can reach any point in the ocean if the fluid velocity is incompressible, bounded, and has small mean drift.

**Zsolt Pajor-Gyulai** *Diffusive transport along heteroclinic networks.* A heteroclinic network in a dynamical system is the union of several hyperbolic equilibrium points and the trajectories connecting them. They arise in multiple contexts, in particular for us, on the boundaries of arrays of recirculating eddies in planar fluid flows. We study the random motion of a passive tracer molecule in such flows and
explore the fundamental effect the presence of a heteroclinic network has on the intermittent behavior of the tracer on intermediate (shorter than homogenization) timescales.

**Yakov Pesin** *Path connectedness of the space of hyperbolic ergodic measures.*

In 1977 Sigmund in a short article proved that for an Axiom A diffeomorphisms the space of ergodic measures is path connected. Following his footsteps one can show that this result holds for the space of hyperbolic ergodic measures for a general diffeomorphism on its isolated homoclinic classes. The latter should be viewed as basic structural elements of any dynamics. Examples will be discussed. This is a joint work with A. Gorodetsky.

**Anthony Quas** *Continuity of Lyapunov exponents for some infinite-dimensional cocycles.*

We study a class of cocycles of compact operators on Hilbert space, and show that under small Gaussian perturbations, the exponents approximate the exponents of the unperturbed cocycle. We also establish the convergence in probability of the Oseledets subspaces.

**Victoria Sadovskaya** *Cohomology, fiber bunching, and Lyapunov exponents of operator-valued cocycles.*

We consider Holder continuous cocycles over hyperbolic systems with values in the group of invertible bounded linear operators on a Banach space. We show that two fiber bunched cocycles are Holder continuously cohomologous if and only if they have Holder conjugate periodic data. The fiber bunching condition means that non-conformality of the cocycle is dominated by the expansion and contraction in the base system. We show that this condition can be established based on the periodic data of the cocycle. This relies on our recent result with B. Kalinin on approximation of the upper and lower Lyapunov exponents of a cocycle in terms of its periodic data.

**Andrew Török** *Central limit theorems for sequential and random intermittent dynamical systems.*

We establish self-norming central limit theorems for non-stationary time series arising as observations on sequential maps possessing an indifferent fixed point (and polynomial decay of correlations). These transformations are obtained by perturbing the slope in the Pomeau-Manneville map. We also obtain quenched central limit theorems for random compositions of these maps. Joint work with M. Nicol and S. Vaienti.

**Ke Zhang** *Random Lagrangian systems and Hamilton-Jacobi equations.*

We show that for a class of random Lagrangian systems, there is a unique, non-uniformly hyperbolic globally minimizing orbit. The hyperbolicity of this orbit is then used to obtain information of the Hamilton-Jacobi PDE, such as regularity of the solution, and speed of convergence to the stationary solution. Based on joint works with K. Khanin and R. Iturriaga.

**Student talks.**

**Changguang Dong** (Penn State) *On density of infinite subsets: lattice action on torus.*

We introduce two types of density property for an infinite subset. One is simply called dense iteration, that is, under a series of iterations, the images of the subset gradually become dense. The other one is quantitative density, which measures the density of the orbits of the infinite subset. Among those results, we will prove that, for any infinite subset of the torus, its images under a sequence of toral automorphisms will become dense. This is, in particular, a strengthen of a previous known result about uniform dilation.

**Alena Erchenko** (Penn State) *A flexibility result for expanding maps on S^1.*

We show that the Lyapunov exponents of a smooth expanding map of degree n on the circle with respect to the measure of maximal entropy and the absolutely continuous invariant measure can take on all possible pairs of values except for well-known restrictions. We will also mention some related problems and further directions in higher dimensions.

**Pratima Hebbar** (Maryland) *Multi-type branching processes with time-dependent branching rates.*
In this talk I will introduce Branching Processes and give some background on the limiting behavior of single type branching processes in the super critical, critical and sub critical cases. I will then introduce multi-type branching processes and explain the results we obtained for these processes with time-dependent branching rates. Our first result gives a criterion for almost sure extinction of the process in terms of the asymptotic behavior of the branching rates, and our second result gives a necessary and sufficient condition for the existence of an exponential limit in the case of a scaled multi-type branching process conditioned on its survival. This is joint work with Dmitry Dolgopyat, Leonid Koralov, and Mark Perlman.

Wenyu Pan (Yale) Local mixing and invariant measures for horospherical subgroups on abelian covers.
It follows from the virtually positive Betti number conjecture (which is now a theorem) that abelian covers of hyperbolic 3-manifolds are ubiquitous. We will discuss the local mixing property of the frame flow on such abelian covers. We will also discuss applications to classification for measures invariant under the horospherical subgroups as well as to counting and equidistribution problems. This is joint work with Hee Oh.

Farruh Shahidi (Penn State) Decay of Correlations for the Katok map.
The Katok map is a slowdown of a linear Anosov map near the origin and it is a local perturbation. It is known that the Katok map is non-uniformly hyperbolic smooth map of the two dimensional torus and is mixing (even Bernoulli) with respect to the area. We obtain polynomial (lower and upper bounds) decay of correlations for this map with respect to the area. We also establish the Central Limit Theorem and Large Deviations for the Katok map (joint with Ya.Pesin and S.Senti).

Yuki Takahashi (Irvine) On a class of Cantor sets that appear in dynamics but are not dynamically defined.
We discuss a class of Cantor sets which are not dynamically defined but appear naturally in partially hyperbolic dynamics and share many properties with dynamically defined Cantor sets. We then consider sums of those Cantor sets, and show that for certain parameter regions it contains both intervals and Cantor sets. The question is motivated by the spectral theory of two dimensional quasicrystals. This is a joint work with J. Fillman and W. Yessen.

Agnieszka Zelerowicz (Penn State) Characterization of equilibrium measures on hyperbolic sets via Caratheodory construction.
Consider a topologically transitive compact locally maximal hyperbolic set $\Lambda$ for an embedding $f : U \to M$ and a Hölder continuous potential function $\phi : \Lambda \to \mathbb{R}$. It is known that there exists a unique equilibrium measure $\mu$ for $\phi$. In my talk I will focus on conditional measures generated by $\mu$ on local unstable manifolds. I will show that these conditional measures can be characterized as Caratheodory measures corresponding to a certain Caratheodory dimension structure. Then I will present a new geometric approach for constructing equilibrium measures using Caratheodory structures. This is joint work with Vaughn Climenhaga and Yakov Pesin.

Adam Zydney (Penn State) Dynamics of Complex Continued Fractions.
Continued fractions of real numbers, both in number theory and in dynamics, are well-understood, but there is relatively little research into complex continued fractions, especially from a dynamics perspective. This talk summarizes some basic facts about complex continued fractions and explores the dynamics of the natural extension map associated to a specific complex continued fraction algorithm.