Math 241 - Calculus I - Sections 02**

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Check list for Test 1

Disclaimer: This list of formulae is only intended to assist you in learning the material. You are supposed to master the material in the book, and if a formula in the book is not included in this list, then it does **not** mean that it is not going to be on quizzes, tests, or the final!

Chapter 2 and Section 3.1

Section 2.1 *Informal discussion of limits.* Calculus is everything that is related to limits. The limit

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

is the cornerstone of Calculus! Applications: tangent lines (geometry) and velocities (physics).

Section 2.2 *Definition of limit.* Let f be defined at each point of some open interval containing a, except possibly at a itself. Then a number L is the limit of $f(x)$ as x approaches a (or is the limit of f at a) if for every number $\epsilon > 0$ (output tolerance, give to us) there exists a number $\delta > 0$ (input tolerance, our friend) such that

$$|f(x) - L| < \epsilon \quad \text{if} \quad 0 < |x - a| < \delta.$$

We can guarantee that the distance of $f(x)$ to L is smaller than any given output tolerance $\epsilon > 0$ if we choose the input tolerance $\delta > 0$ small enough.

Important formulae: Equation of the tangent line to the graph of f at $(a, f(a))$,

$$y = m_a(x - a) + f(a), \qquad m_a = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

Position of an object under the influence of gravity:

$$h(t) = -\frac{1}{2} g t^2 + v_0 t + h_0,$$

where g is the gravitational constant. Watch for the units!

Section 2.3 *Limit theorems and continuity.* Sum rule, constant multiple rule, difference rule, product rule, quotient rule.

A function f is *continuous* at a number a in its domain if

$$\lim_{x \to a} f(x) = f(a).$$

"The limit is given by plugging in $x = a$." A function f is discontinuous at a number a in its domain if f is not continuous at a.

Section 2.4 *The squeezing theorem and substitution rule.*

The squeezing theorem is a *geometric* fact: If $f(x) \leq g(x) \leq h(x)$ for all x in some open interval about a except possibly a itself, and if

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L,$$

then

$$\lim_{x \to a} g(x) = L.$$

Important limits:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1, \qquad \lim_{x \to 0} \frac{\cos x - 1}{x} = 0, \qquad \lim_{x \to 0} \frac{e^x - 1}{x} = 1.$$

The substitution rule is *conceptually* important, it allows us to reduce complicated situations to a sequence of easier ones. The idea is the following: If

$$\lim_{x \to a} f(x) = c, \quad \text{and} \quad \lim_{y \to c} g(y) \text{ exists,}$$

then

$$\lim_{x \to a} g(f(x)) = \lim_{y \to c} g(y).$$

It is important that you know how to identify the *outer* function g and the *inner* function f.

Section 2.5 *One-sided and infinite limits.* Make sure you understand what the notation

$$\lim_{x \to a^-} f(x), \qquad \lim_{x \to a^+} f(x)$$

means. It is an important fact that the limit of f exists at a if

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x).$$

In this case

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = \lim_{x \to a} f(x).$$

You should also know how to determine whether the limit of $f(x)$ as x approaches a from the left or the right is ∞ or $-\infty$. Other important definitions: The line $x = a$ is a vertical asymptote of the graph of f. The graph of f has a vertical tangent line at $(a, f(a))$ if f is continuous and the cornerstone limit is infinite at a.

Section 2.6 *Continuity on intervals and the intermediate value theorem.* You should know how to determine the intervals on which f is continuous. The intermediate value theorem guarantees that a continuous function attains all values between $f(a)$ and $f(b)$ on the interval $[a, b]$. In particular, we can *solve the equation* $f(x) = 0$ if $f(a)$ and $f(b)$ have different signs. An algorithm is the bisection method.

Section 3.1 *The derivative.* Let a be a number in the domain of a function f. If

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

exists, we call this limit the derivative of f at a and write it $f'(a)$. Note that $f'(a)$ is the slope of the tangent line to the graph of f at $(a, f(a))$.