Math 241 - Calculus I - Sections 02**

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Check list for Test 2

Disclaimer: This list of formulae is only intended to assist you in learning the material. You are supposed to master the material in the book, and if a formula in the book is not included in this list, then it does **not** mean that it is not going to be on quizzes, tests, or the final!

Chapter 3

Section 3.1 *The derivative.* Let a be a number in the domain of a function f. If

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

exists, we call this limit the derivative of f at a and write it $f'(a)$. Note that $f'(a)$ is the slope of the tangent line to the graph of f at $(a, f(a))$.

Section 3.2 *Differentiable functions.* If a function f is differentiable at each number in its domain, then f is a differentiable function. Examples

$$\frac{d}{dx}\, x^n = nx^{n-1}, \quad \frac{d}{dx} \cos x = -\sin x, \quad \frac{d}{dx} \sin x = \cos x, \quad \frac{d}{dx}\, e^x = e^x.$$

Section 3.3 *Derivatives of combinations of functions.* Important rules to remember: the sum/difference rule, the constant multiple rule, the product rule

$$(fg)'(a) = f'(a)g(a) + f(a)g'(a),$$

the quotient rule

$$\left(\frac{f}{g}\right)'(a) = \frac{f'(a)g(a) - f(a)g'(a)}{(g'(a))^2}.$$

Section 3.4 *The chain rule.* One way to remember that the chain rule gives you a product of two derivatives is to say that

chain rule = (outer derivative) x (inner derivative).

Mathematically

$$(g \circ f)'(a) = g'\big(f(a)\big)f'(a).$$

Section 3.5 *Higher derivatives.* The nth derivative $f^{(n)}(a)$ can be found by differentiating the $(n-1)$st derivative,

$$f^{(n)}(a) = \lim_{x \to a} \frac{f^{(n-1)}(x) - f^{(n-1)}(a)}{x - a}.$$

Application: the acceleration of an object is the second derivative of the function giving its position (as a function of time).

Section 3.6 *Implicit differentiation.* How to find the derivative of y if y is not given explicitly as $y = f(x)$ but implicitly in the form of an equation that involves y and x? Differentiate both sides of the equation with respect to x and use the chain rule to find the derivatives of terms that involve y (that is a function of x).

Application: Find the tangent line to the graph of a function that is given implicitly.

Section 3.7 *Related rates.* It is important that you carefully follow the steps to solve problems with related rates:

1) Read the problem carefully, make a sketch, identify and label the different variables.

2) Find an equation that relates the variables. Typically this involves geometric facts such as similar triangles, the Pythagorean Theorem or trigonometric identities.

3) Differentiate the equation implicitly to find an equation relating rates.

4) Solve for the rate you need to find.

Section 3.8 *Approximation.* Linear Approximation: we evaluate the tangent line instead of the function f. This leads to

$$f(a+h) \sim f(a) + hf'(a).$$

The Newton-Raphson Method: We try to find a zero of a function by replacing the function with the tangent line. The zero of the tangent line can be found easily (if it is not parallel to the x-axis). This leads to

$$c_n = c_{n-1} - \frac{f(c_{n-1})}{f'(c_{n-1})}$$

or

$$c_n = g(c_{n-1})$$

where

$$g(x) = x - \frac{f(x)}{f'(x)}.$$