

MATH 406 – HOMEWORK I

(due Wednesday 4 February 2009)

1. Assume that $a, b \in \mathbb{Z}$ and $b \neq 0$. Let $S = \{a + bk \mid k \in \mathbb{Z}, a + bk > 0\}$. Show that S has a smallest element but no largest element.
2. Find integers a, b with $1 \leq a \leq 6$ such that $|a\sqrt{5} - b| < 1/6$. (You may use a calculator)
2. Prove that $[x] + [x + 1/2] = [2x]$ for all real numbers x .
4. Use mathematical induction to prove that $n < 2^n$ for all positive integers n .
5. Use (strong) mathematical induction to show that every integer $n \geq 8$ can be written as $n = 3a + 5b$ for non-negative integers a, b .

NOTE: Explain your work clearly. Your solutions must include enough detail to justify your conclusions.