MATH 406 – HOMEWORK XI

(due Friday 8 May 2009)

1. Show that if x, y, z is a primitive Pythagorean triple then either x or y is divisible by 3.

2. Find formulas for the integers of all (primitive) Pythagorean triples x, y, z with $z = y + 1$.

3. Show that the Diophantine equation $x^4 + 3y^4 = z^2$ has infinitely many solutions.

4. Determine which of the following can be written as the sum of two squares.

(a) 65

(b) 99

(c) 999

(d) 1000

5. Show that a positive integer is the difference of two squares iff it does not have the form $4k + 2$.

NOTE: Explain your work clearly. Your solutions must include enough detail to justify your conclusions.