MATH 406 – HOMEWORK II

(due Wednesday 11 February 2009)

1. Use (strong) induction to show that $f_{n+3} - f_n = 2f_{n+1}$ for all $n \in \mathbb{Z}^+$.

2. Show that f_n is even iff $3|n$.

('iff' means 'if and only if'.)

3. Show that $3|(k^3 - k)$ for all $k \in \mathbb{Z}^+$.

4. Find the number of positive integers ≤ 1000 which are not divisible by 3 and not divisible by 5. Explain carefully! There will be no credit given for listing and counting them directly. Hint: recall the result in problem 21 in section 1.5.

5. Prove that for every integer $n \geq 2$, $n^3 + 1$ is not prime. (Hint: $n = k + 1$ for some $k \in \mathbb{Z}^+$.)

NOTE: Explain your work clearly. Your solutions must include enough detail to justify your conclusions.