

## MATH 406 – HOMEWORK VI

*(due Wednesday 25 March 2009)*

1. Determine the remainder when  $2 \cdot 4 \cdot 6 \cdot 8 \cdot 10 \cdot 12 \cdot 14 \cdot 16 \cdot 18 \cdot 20 \cdot 22 \cdot 24$  is divided by 13. Explain.
2. Determine the remainder when  $6^{2009}$  is divided by 11. Explain.
3. Find the last digit of the decimal expansion of  $7^{1,000,001}$ .
4. Use Euler's Theorem to solve  $5x \equiv 3 \pmod{16}$ .
5. Let  $a, b$  be positive integers with  $(a, b) = 1$ . Show that  $a^{\phi(b)} + b^{\phi(a)} \equiv 1 \pmod{ab}$ .

**NOTE:** Explain your work clearly. Your solutions must include enough detail to justify your conclusions.