

MATH 406 – HOMEWORK VII

(due Wednesday 1 April 2009)

1. Determine all positive integers n with $\phi(n) = 14$. Explain.
2. Show that $\phi(m^k) = m^{k-1}\phi(m)$ for all positive integers m, n .
3. Determine all positive integers n for which $\sigma(n)$ is odd. Explain.
4. Let $m \geq 2$, let $n = 2^{m-1}(2^m - 1)$, and assume that $2^m - 1$ is not prime. Show that $\sigma(n) > 2n$.
5. Determine the prime power factorization of M_{15} . Explain, and avoid brute force. Note that a result we proved gives two factors.

NOTE: Explain your work clearly. Your solutions must include enough detail to justify your conclusions.