

## MATH 406 – HOMEWORK IX

*(due Friday 24 April 2009)*

1. [4] Prove Euler's Criterion (Theorem 11.3) using indices and Theorem 11.2.
2. [4] Use Gauss' Lemma to evaluate  $\left(\frac{5}{17}\right)$ .
3. [6] Use the arithmetic properties of the Legendre symbol to evaluate the following:
  - (a)  $\left(\frac{-8}{17}\right)$ .
  - (b)  $\left(\frac{77}{103}\right)$ .
4. [6] Let  $p$  and  $q$  be odd primes and assume that  $p \equiv q \pmod{20}$ . Prove that  $\left(\frac{5}{p}\right) = \left(\frac{5}{q}\right)$ .

**NOTE:** Explain your work clearly. Your solutions must include enough detail to justify your conclusions.