Simultaneous Calibration and Nonresponse Adjustment

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OUTLINE

I. Def'n of Constrained Optimum Problem for Loss plus Penalty
— loss for 2 types of adjustment (cf Deville & Särndal 1992)

II. Form of Linear Single-Stage Equations & Iterative Sol'ns
— comparison with special & limiting Cases

III. Superpopulation Properties of Solutions
— consistency & linearized variance formulas

IV. Numerical Illustration with SIPP 96 Data

Notations & Formulation

Frame $\mathcal{U}$, Sample $\mathcal{S}$, Initial (inclusion) weights $w_k^o = \frac{1}{\pi_k}$
Unit response indicators $r_k = 0, 1$, observe $(y_k : \ k \in \mathcal{S}, \ r_k = 1)$

$\mathbf{x}_k$ nonresponse calibration variables, usually $(I_{[k \in C_1]}, \ldots, I_{[k \in C_m]})$
$\mathbf{z}_k$ population calibration/control variables

fixed 'known' totals $t^*_{\mathbf{x}} \approx \sum_{k \in \mathcal{U}} \mathbf{x}_k$, $t^*_{\mathbf{z}} \approx \sum_{k \in \mathcal{U}} \mathbf{z}_k$

Relaxed (nonresp. adjusted) weights w_k , final weights $\widehat{w}_k$

Survey totals $\sum_{k \in \mathcal{U}} y_k$ estimated by $\widehat{t}_{y,adj} = \sum_{k \in \mathcal{S}} r_k \, \widehat{w}_k \, y_k$

Convex Penalty function $Q(\cdot)$ will be applied to $\widehat{w}_k / w_k^o$

Objective:

Weight-adjustment transitions $r_k \, w_k^o \mapsto w_k \mapsto \widehat{w}_k$

as small as possible, with $\widehat{w}_k / w_k^o$ constrained to be moderate for $r_k = 1$

Then estimate survey totals $\sum_{k \in \mathcal{U}} y_k$ by

$$\widehat{t}_{y,adj} = \sum_{k \in \mathcal{S}} r_k \, \widehat{w}_k \, y_k$$

Motivation

- Current methods usually start with nonresponse weight adjustment, then impose population controls (eg by raking to demographic-cell census counts) with some weight trimming.

- Methods based on linearization, HT variance formulas require joint inclusion probabilities, but these are available only *before* weight adjustments !

- Ambiguous role of nonresponse adjustment: are the demographic cell proportions to be maintained or not ?

Single Stage Weight Adjustment Method

- New approach does all adjustments in **single stage**

$$\min_{\mathbf{w},\,\hat{\mathbf{w}}} \sum_{k\in\mathcal{S}} r_k\, w_k^o \left\{ G_1\left(\frac{w_k - w_k^o}{w_k^o}\right) + \alpha\, G_2\left(\frac{\hat{w}_k - w_k}{w_k^o}\right) + Q\left(\frac{\hat{w}_k}{w_k^o}\right) \right\}$$

$$\text{subject to:} \quad \sum_{k\in\mathcal{S}} r_k\, w_k\, \mathbf{x}_k = t^*_{\mathbf{x}} \quad , \quad \sum_{k\in\mathcal{S}} r_k\, \hat{w}_k\, \mathbf{z}_k = t^*_{\mathbf{z}}$$

w_k are approx. nonresp-adjusted weights, $\hat{w}_k$ are **final weights**

$$G_1(z),\ G_2(z)\ \textit{loss functions} = \begin{cases} z^2/2 & \text{for linear calibration} \\ z\log z - z + 1 & \text{for raking} \end{cases}$$

$Q(z)$ convex penalty function $\equiv 0$ on interval , e.g. $[.6, 2]$
e.g. finite only on $(.1, 10)$

Equations for Lagrange Mult's & $\widehat{w}_k$ when $G_j(z) \equiv z^2/2$

$$\widehat{w}_k + \frac{1+\alpha}{\alpha} w_k^o \, Q'(\frac{\widehat{w}_k}{w_k^o}) \;=\; w_k^o \, \{1 + \frac{1+\alpha}{\alpha} \mu' \mathbf{z}_k + \lambda' \mathbf{x}_k\}$$

Lagrange multipliers λ, μ for $t^*_{\mathbf{x}}, t^*_{\mathbf{z}}$ constraints determined by

$$M_\alpha \;=\; \sum_{k \in \mathcal{S}} r_k \, w_k^o \begin{pmatrix} \mathbf{x}_k \, \mathbf{x}_k' & \mathbf{x}_k \, \mathbf{z}_k' \\ \mathbf{z}_k \mathbf{x}_k' & (1+\alpha^{-1}) \, \mathbf{z}_k \, \mathbf{z}_k' \end{pmatrix}$$

$$M_\alpha \binom{\lambda}{\mu} \;=\; \binom{t^*_{\mathbf{x}}}{t^*_{\mathbf{z}}} - \sum_{k \in \mathcal{S}} r_k \, w_k^o \binom{\mathbf{x}_k}{\mathbf{z}_k} + \sum_{k \in \mathcal{S}} r_k \, w_k^o \, Q'(\frac{\widehat{w}_k}{w_k^o}) \binom{\mathbf{x}_k}{(1+\frac{1}{\alpha}) \, \mathbf{z}_k}$$

NB. $\widehat{w}_k - w_k \;=\; \frac{w_k^o}{\alpha} (\mu' \mathbf{z}_k \; - \; Q'(\widehat{w}_k / w_k^o))$ `small when` α `large`

Comparison with Standard ‘Two-Stage Method’

$\mathbf{x}_k \;=\; (I_{[k\in C_1]}, \ldots, I_{[k\in C_m]})$ indicators of *adjustment cells*

$\mathbf{z}_k$ totals for population controls

1st stage: ratio adjust, for $i \in C_j$, $w_i \equiv r_i\, w_i^o \, \frac{\sum_{\mathcal{S}} I_{[k\in C_j]}}{\sum_{\mathcal{S}} r_k\, I_{[k\in C_j]}}$

2nd stage: calibrate $\widehat{w}_k \;=\; (\sum_{\mathcal{S}} w_k\, \mathbf{z}_k\, \mathbf{z}_k')^{-1}\, (t^*_{\mathbf{z}} - \sum_{\mathcal{S}} w_k\, z_k)$

Notes: (i) in US, often do 2nd stage by raking instead,

(ii) inclusion prob. based variance formula given in Särndal & Lundström (2005, Sec. 11.4)

(iii) consistent (Fuller 2002) when adjustment cell response model (ρ_k constant for k in each C_j) is correct and $t^*_{\mathbf{z}}$ is exact frame **z** total.

Advantages of New Method

- Ease of documentation of adustment/controls/trimming

- Tuning parameters (α and const's in Q)
  - — small α roughly approximates '2-stage method'
  - — large α method gives ($w_k \approx \widehat{w}_k$ and) simultaneous one-stage calibration to $(\mathbf{x}_k, \mathbf{z}_k)$.

- Variance formulas (not shown), based on inclusion probabilities & linearization, for given α and Q

- New method does not dramatically change estimates but allows approx. calibration to enhance consistency when some calibration totals are off.

Superpopulations and Large-Sample Consistency

`Superpopulation model:` sequence of large frames, $N = |\mathcal{U}| \to \infty$

Populations non-random, r_k indep. $\{0, 1\}$ resp. indicators

Design and r model Consistency for survey est's based on 2-stage *and* 1-stage methods when

(i) $1/P(r_k = 1) \equiv 1 + \lambda' \mathbf{x}_k$, (ii) lim $\frac{\sqrt{n}}{N} \begin{pmatrix} t^*_{\mathbf{x}} - \sum_{\mathcal{U}} \mathbf{x}_k \\ t^*_{\mathbf{z}} - \sum_{\mathcal{U}} \mathbf{z}_k \end{pmatrix}$ bdd.

(iii) $Q(1 + \lambda' \mathbf{x}_k) = 0$ for all but a negligible fraction of $k \in \mathcal{U}$.

But otherwise: consistency depends on (ii) along with 'model-based' assumption that for some β the residuals

$$y_k - \beta' \mathbf{z}_k \quad \text{(or } \; y_k - \beta' \binom{\mathbf{x}_k}{\mathbf{z}_k} \; \text{ when } \alpha \text{ large)}$$

are approx. orthogonal to all lin. comb's of $\mathbf{x}_k$ or $\binom{\mathbf{x}_k}{\mathbf{z}_k}$ entries.

SIPP Background

Stratified national Survey on Income & Program Participation

217 Strata, of which 112 are Self-Representing (SR)

Longitudinal design, consider only 94444 Wave 1 responders.

Nonresponse adjustment by 149 demographic & economic Adjustment Cells; here $t^*_{\mathbf{x},j} = N \sum_{C_j \cap \mathcal{S}} w^o_k / \sum_{\mathcal{S}} w^o_k$.

Population controls to updated census estimates of 126 linearly independent demographic-cell population totals.

Numerical Results from Adjusting SIPP 96 Weights

Multipliers from alpha=1 Adjustments

	Min.	Q1	Median	Mean	Q3	Max.
Lambda	0.079	0.177	0.216	0.200	0.231	0.256
Mu	-1.126	-0.025	0.048	0.032	0.114	0.723
Final/Base	0.681	1.097	1.184	1.212	1.285	3.773

Multipliers from alpha=100 Adjustments

	Min.	Q1	Median	Mean	Q3	Max.
Lambda	-0.355	-0.052	0.292	0.171	0.335	0.404
Mu	-0.859	0.028	0.187	0.174	0.379	1.773
Final/Base	0.622	1.093	1.193	1.208	1.298	4.076

Correlation between Calibrated (pop-controlled) and new weights is : 0.995 for $\alpha = 1$ and 0.968 for $\alpha = 100$.

Estimated Totals & Std Dev's, 2stg & 1stg Methods
Totals and SD's in 1000's, all SIPP 96 Strata

	Totals			Std Errors		
Item	2stg	$\alpha = 1$	$\alpha = 100$	VPLX	$\alpha = 1$	$\alpha = 100$
FOODST	27541	27454	26930	687	318	301
AFDC	14123	14089	13800	450	298	288
MDCD	28468	28399	27895	574	404	351
SOCSEC	36994	37071	37240	470	157	157
HEINS	194216	194475	195030	1625	439	423
POV	41951	41978	41475	747	360	357
EMP	190871	190733	190097	1477	255	240
UNEMP	6403	6379	6295	163	144	143
NILF	66979	67354	67864	627	231	217
MAR	111440	114457	114347	1088	159	158
DIV	18534	18542	18591	253	195	195

1-Stg SE's reflect assumed **known** nonresp. adj. totals $t^*_{\mathbf{x}}$

Discussion / Summary

- New single-stage weight adjustment methods allow design-consistent survey estimators on same footing as standard two-stage methods when calibration totals and nonresponse model are valid.

- Incorporating penalty function for large & small weights into the single-stage adjustments enhances numerical stability when calibration totals and nonresponse mode are incorrect.

- The new methods with large α allow slightly greater 'model-based' protection against incorrect model and pop-controls.

- Generalization to other G_j loss-functions including those associated with raking is a topic of further research.

References

- Chang, T. and Kott, P. (2008), *Biometrika*
- Deville, J.-C. and Särndal, C.-E. (1992), Calibration estimators in survey sampling, *JASA*.
- Fuller, W. (2002), Regression estimation for survey samples. *Surv. Meth.*
- Oh, H. and Scheuren, F. (1983) Weighting adjustment for unit nonresponse.
- Särndal, C.-E. and Lundström, S. (2005) *Estimation in Surveys with Nonresponse*: Wiley.
- Singh, A. and Mohl, C. (1996), *Surv. Meth.*.
- Théberge, A. (2000) Calibration and restricted weights. *Surv. Meth.*