Math 406 – Fall 2025 – Harry Tamvakis

PROBLEM SET 2 – Due September 18, 2025

Reading for this week: Section 1.

Problems

From the book: Appendix A, Problem #8; Section 1, Problems #2, 4, 6, 8, 10, 14. In addition, do the following problems:

A1) (a) Prove that the square of any integer is either of the form $3k$ or $3k+1$.

(b) Prove that the cube of any integer has one of the forms: $9k$, $9k+1$, or $9k+8$.

(c) Show that there exist integers which cannot be written as a sum of three cubes. For example, verify that there do not exist integers a, b, and c (possibly negative) such that $a^3 + b^3 + c^3 = 5$.

A2) (a) How many natural numbers less than or equal to 1000 are divisible by 3? Divisible by 5? Divisible by 7?

(b) How many natural numbers less than or equal to 1000 are divisible by 3 or by 5?

(c) How many natural numbers less than or equal to 1000 are divisible by 3 or by 5 or by 7?

A3) Find all triples (a, b, c) of positive integers with $a < b < c$ such that the sum of any two of them is divisible by the third.

Extra Credit Problems.

EC1) Section 1, Problem 15.

EC2) Suppose that a, b, and c are rational numbers such that $a + b + c$, $ab + bc + ca$, and abc are integers. Prove that a, b, and c are all integers.