**Math 406 – Fall 2025 – Harry Tamvakis**

**PROBLEM SET 4 – Due October 2, 2025**

Reading for this week: Sections 3 and 4 (except for Theorem 6 on page 31).

## Problems

From the textbook: Section 3, Problems #2, 4, 5, 6; Section 4, Problems #2, 3, 4, 6, 19. In addition, do the following problems:

**A1)** Let $\tau(n)$ be the number of positive divisors of $n$. Show that $\tau(n) = \tau(n+1) = \tau(n+2) = \tau(n+3)$ if $n = 3655$.

**A2)** An old receipt has faded. It reads 88 chickens at a total of $\$\, x4.2y$, where $x$ and $y$ are unreadable digits. How much did each chicken cost?

**A3)** (a) Prove that the integer $111^{333} + 333^{111}$ is divisible by 7.

(b) Prove that the integer $53^{103} + 103^{53}$ is divisible by 39.

## Extra Credit Problems.

**EC1)** Use congruences to find the last two digits of $2^{100}$.

**EC2)** A positive integer is called *polite* if it can be represented as a sum of two or more consecutive positive integers. For example, 7 and 22 are polite since $7 = 3 + 4$ and $22 = 4 + 5 + 6 + 7$, while 2 is impolite. Prove that the only impolite positive integers are the powers of 2, that is, 1, 2, 4, 8, 16, ...