## Math 406 – Fall 2025 – Harry Tamvakis PROBLEM SET 6 – Due October 23, 2025

Reading for this week: Sections 6, 7, and 8.

## **Problems**

From the textbook: Section 6, #11, 12, 19, 20, Section 7, #12, 16, Section 8, #2, 5, 7, 13. In addition, do the following problems:

- **A1)** From Fermat's theorem deduce that 13 divides  $11^{12n+6} + 1$  for all nonnegative integers n.
- **A2)** a) Find the remainder when 13! is divided by 17.
- b) Find the remainder when 2(28!) is divided by 31.

## Extra Credit Problem.

**EC)** A complete system of residues modulo n is a set of n numbers such that no two of them are congruent modulo n. Let p be an odd prime and let  $a_1, \ldots, a_p$  and  $b_1, \ldots b_p$  be complete systems of residues modulo p. Prove that  $a_1b_1, \ldots, a_pb_p$  is not a complete system of residues modulo p.