Math 660 – Spring 2024 – Harry Tamvakis
PROBLEM SET 9 – Due April 18, 2024

1) Exercise 284 in the textbook.

2) Let Ω be a domain in $\mathbb{C}$ and $\{a_n\}$ be a sequence of distinct points in Ω with no limit point in Ω. For each integer $n \geq 1$, choose integers $k_n \geq 0$ and constants $c_{n,k}$ for $0 \leq k \leq k_n$. Prove that there exists a holomorphic function f on Ω such that $f^{(k)}(a_n) = c_{n,k}$, for all n and k with $0 \leq k \leq k_n$.

3) Let Ω be an open set in $\mathbb{C}$ and $\{D_i\}_{i \in I}$ be an open cover of Ω by disks D_i. For each index $i \in I$, we are given a meromorphic function h_i on D_i, not identically zero. Assume that for all indices $i, j \in I$, the function $g_{ij} := h_i/h_j$ is holomorphic on $D_i \cap D_j$. Prove that for all indices $i \in I$, there exists a holomorphic function f_i on D_i without any zero and such that $f_i = g_{ij} f_j$ on $D_i \cap D_j$, for all $i, j \in I$.

4) Exercise 295 in the textbook.

5) Let Ω be a bounded domain in $\mathbb{C}$, and denote the group of biholomorphic maps of Ω into itself by $\mathrm{Aut}(\Omega)$. Let $a \in \Omega$ and let $\mathrm{Aut}_0(\Omega) = \{f \in \mathrm{Aut}(\Omega) \mid f(a) = a\}$. Prove that, with the topology of uniform convergence on compact subsets of Ω, $\mathrm{Aut}_0(\Omega)$ is *compact*.