

## Laplace Transform Table

Function	Laplace Transform
$t^n, n = 0, 1, \dots$	$n! / s^{n+1}$
$t^n, n > -1$	$\Gamma(n+1) / s^{n+1}$
$e^{at}$	$1 / (s - a)$
$t^n e^{at}$	$n! / (s - a)^{n+1}$
$e^{at} \cos(\omega t)$	$(s - a) / ((s - a)^2 + \omega^2)$
$e^{at} \sin(\omega t)$	$\omega / ((s - a)^2 + \omega^2)$
$e^{at} (\sin(\omega t) - \omega t \cos(\omega t))$	$2\omega^3 / ((s - a)^2 + \omega^2)^2$
$te^{at} \sin(\omega t)$	$2\omega(s - a) / ((s - a)^2 + \omega^2)^2$
$f'(t)$	$s\mathcal{L}(f) - f(0)$
$f''(t)$	$s^2\mathcal{L}(f) - sf(0) - f'(0)$
$f^{(n)}(t)$	$s^n\mathcal{L}(f) - s^{n-1}f(0) - \dots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
$e^{at} f(t)$	$\mathcal{L}(f)(s - a)$ , i.e., plug in $s - a$ for $s$ .
$tf(t)$	$-\mathcal{L}(f)'$
$t^n f(t)$	$(-1)^n \mathcal{L}(f)^{(n)}$
$\int_0^t g(t-u)f(u) du = f * g$	$\mathcal{L}(f)\mathcal{L}(g)$
$H_c(t)f(t-c)$	$e^{-cs}\mathcal{L}(f)$
$\delta(t-c)f(t)$	$e^{-cs}f(c)$
$f(t)$	$\int_0^\infty e^{-st}f(t) dt$

Note that the variable  $s$  and the function  $f$  can be complex valued. Thus the results for trig functions actually follow from those for exponential functions.

The gamma function above is  $\Gamma(x) = \int_0^\infty x^{n-1}e^{-x} dx$ . It satisfies  $\Gamma(x+1) = x\Gamma(x)$  and  $\Gamma(n+1) = n!$  for  $n$  a positive integer. Also  $\Gamma(1/2) = \sqrt{\pi}$ .