MATH 115 Section 1.5
Three types of equations which you should feel comfortable solving.

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1. Equations involving radicals. Typically these equations involve one (usually) or more (rarely) square roots.

   Example: $\sqrt{3-x}+15=x+12$

   The method of solution is straightfoward. The way to think about this is by asking the question "What's the thing you like the least?" The students invariably reply "The square root" so then you can explain that the goal is to get rid of it. We therefore have the following steps:

   (a) Isolate the radical (square root).
   Example: In our example this gives us $\sqrt{3-x}=x-3$.

   (b) Square both sides. Remember that sometimes (but not always) this means FOILing on one side.
   Example: In our example this gives us $\left(\sqrt{3-x}\right)^2=(x-3)^2$ and hence $3-x=x^2-6x+9$.

   (c) Solve as usual. Sometimes this requires factoring or the quadratic formula.
   Example: In our example rewrite as $x^2-5x+6=0$ which has solutions $x=2$ and $x=3$ (make sure you show the steps to the final answer.)

   (d) We think we're done, but for some equations sometimes solutions arise in the solving process which are fake, or ghost solutions. Officially these are *extraneous solutions* which is a misnomer because they're not solutions at all. For this type of equation we must check the solutions in the original problem.
   Example: In our example we try and see $x=2$ does not work but $x=3$ does. Therefore $x=2$ is thrown away and $x=3$ is the solution.

   Example: Another good example (if time permits) is $\sqrt{2x+1}=\sqrt{5x}$.

2. Equations involving rational (fractional) expressions. These equations involve fractions which have variables in the denominator.

   Example: $\frac{4}{x-1}+\frac{5}{x+1}=\frac{42}{x^2-1}$

   Sometimes people learn a variety of ways to solve these but the guaranteed single approach is:

   (a) Find the greatest common denominator. Sometimes we will be required to factor a denominator or two.
   Example: In our example we have $x^2-1=(x-1)(x+1)$ and so the GCD is $(x-1)(x+1)$ because it contains all denominators.

   (b) Multiply both sides through by the GCD. If you are careful your result will have no denominators whatsoever. Remember in the first type of equation we eliminated the thing we hated the most and we are doing the same thing here by eliminating the denominators.
   Example: In our example if we're careful we get the new equation $4(x+1)+5(x-1)=42$

   (c) Solve.
   Example: In our example we get $x=\frac{43}{9}$.

   (d) Like the previous type, sometimes extraneous solutions arise but there is a shortcut test here. Simply check if your solution makes any of the denominators of the original problem equal to 0. If it does, throw it out.
   Example: In this case $x=\frac{43}{9}$ does not make any denominator 0 so it's safe and hence a solution.

   Example: Another good example (if time permits) is $2+\frac{3x}{(x+4)(x+2)}=\frac{1}{x+4}+\frac{3}{x+2}$

One more type on reverse side!

3. Equations of quadratic type. These could be both quadratic equations or those that look a bit quadratic-like.

   Example: $x^4 - x^2 = 6$

   Note that if we move the 6 over we get $x^4 - x^2 - 6 = 0$ which is not quadratic but reminds you of a quadratic equation. What can we do to solve this? Usually you'll have a few students who can see that $x^4 = \left(x^2\right)^2$ or you can prompt them to this. The method of solution is then:

   (a) Make a substitution which converts your equation to a quadratic equation. For many students this is their first exposure to substitution so point out it's just an aid to simplification. The choice of substitution is sometimes obvious but will be more obvious after they've struggled through a few examples themselves and found the patterns.

   Example: In our example let $u = x^2$ and so then the equation becomes $u^2 - u - 6 = 0$. Note that u is typically used but any letter other than the original one is okay.

   (b) Solve for u.

   Example: In our example we get $u = 3$ and $u = -2$.

   (c) In the solutions, replace with the x's and then solve for x.

   Example: In our example we get $x^2 = 3$ and $x^2 = -2$. Of these only the first has solutions, $x = \pm\sqrt{3}$.

   Example: Another good example (if time permits) is $\sqrt{x} + 3\sqrt[4]{x} + 2 = 0$ or if they seem really smart replace x by $x+3$ or switch the order of the two first terms which makes the substitution highly nonobvious.