MATH 115 Section 1.6
Modeling with Equations - An Approach to Solving Word Problems

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1. **Introduction:** The types of equations which come up when modeling real-world situations are limitless and we will see many different types in this course. There are also many different ways to solve real-world problems, too. This section is meant to give you a good starting point for when you are stuck with a problem.

2. Here are **four steps** which can help you when you're stumped. We'll list them first and then see how some problems can be solved by applying these steps.

   (a) Identify the variable. What are you trying to find? Give it a variable name, maybe one which is representative (c=cost, t=time or h=height, for example.)

   (b) If there are any other unknown quantities in the problem, express these in terms of your variable from (a). Doing this prevents you from having too many variables which can lead to confusion.

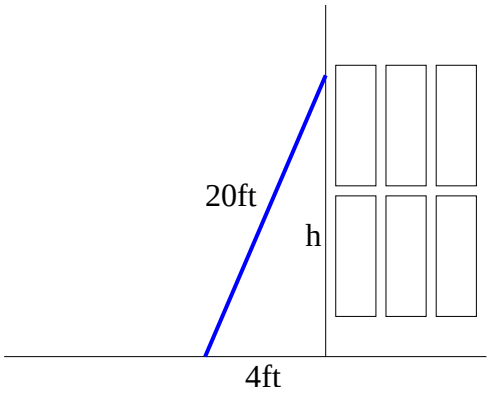
   (c) Set up the model. The problem gives a situation and this situation will translate to an equation involving your variable.

   (d) Solve the equation for your variable. Check your solution *in the original wording of the problem.*

3. Here are a few examples. Don't worry so much about the formulas that arise, focus more on the consistent method which arises to solve the problems.

   Example: A 20-foot ladder leans against a building. The base of the ladder is 4 feet from the building. How high up the building does the ladder go?

   (a) Let's draw a picture and label what we know and want. We see we want the height so let's call it h on the picture.



   (b) There are no other unknowns, so there is nothing to do here.

   (c) The equation which arises here is the Pythagorean Theorem. We know $(4)^2 + h^2 = (20)^2$.

   (d) We can solve this getting $h = \pm\sqrt{384} = \pm 8\sqrt{6}$ and since it's a length we only need the positive value, so $h = 8\sqrt{6} \approx 15.5959$ feet.

Example: Jack and Diane both leave Washington DC heading north on I-95 at the same time. Jack is driving his car at 65mph while Diane is riding her motorcycle at 78mph. How long will it take for Diane to be 20 miles ahead of Jack?

(a) What are we looking for? The time (call it t) at which Diane is 20 miles ahead of Jack.

(b) Are there any other unknowns? There are a few which are relevent to the problem. Let's list them and find them all in terms of t.

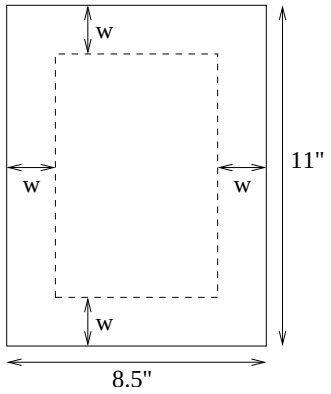
- After time t, Jack has gone $65t$ miles. This is from distance = rate $\times$ time.
- After time t, Diane has gone $78t$ miles.
- After time t, Diane is $78t - 65t$ miles ahead of Jack.

(c) The problem stipulates that we wish to know when Diane is 20 miles ahead of Jack, so we want $78t - 65t = 20$. This is our equation.

(d) We solve and get $t = \frac{20}{13} \approx 1.5384$ hours.

Example: Suppose I start with an $8.5'' \times 11''$ piece of paper. I want to cut away a margin around the outside of uniform width and end up with a sheet having half the area I started with. How wide should my cut margin be?

(a) What are we looking for? We're looking for the width, call it w. A picture might help:



(b) What else don't we know? Well there are lots of things on the picture we don't know, but specifically we don't have the dimensions once we've cut. These are easy though because w comes off each end and so:

- The new length will be $11 - w - w = 11 - 2w$.
- The new width will be $8.5 - w - w = 8.5 - 2w$.

(c) The problem desires the new area to be half the old area, but what are these? Half the old area is easy, it's $\frac{1}{2}(8.5)(11)$ but the new area is easy too, it's $(11 - 2w)(8.5 - 2w)$. Thus we need $(11 - 2w)(8.5 - 2w) = \frac{1}{2}(8.5)(11)$.

(d) Solving this with the Quadratic Formula yields two solutions:

- $w = \frac{39+\sqrt{773}}{8} \approx 8.35''$
- $w = \frac{39-\sqrt{773}}{8} \approx 1.40''$

So which of these makes sense? Make sure they really think about this because many of them will suggest that either one solution should have been negative (and hence thrown out) or that both are valid. The problem with the first of course is that we can't cut this much paper from both sides of a sheet, so only the second is valid.