MATH 115 Section 1.7
Solving Inequalities

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1. **Definition:** An *inequality* is like an equation except the = has been replaced by >, <, ≥ or ≤. We need to know how to deal with two types of these, linear and nonlinear (some types).

2. **Linear Inequalities:** A *linear inequality* is the most basic kind. This is an inequality where both sides can be simplified to the form $ax + b$.

   Example: $3x + 3 \leq -x + 7$

   Example: $2(x + 1) - 3x + 5 > \frac{1}{2}x - 8$

   Solving these is very easy and most students know how to do these already. Solve it like you would a linear equation, except when you multiply or divide by a negative number you must switch the direction of the inequality.

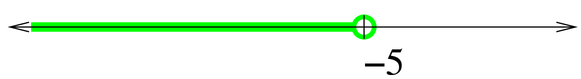
   The other thing that the students need to know how to do is represent the solutions three different ways:

   - With inequalities.
   - On a number line.
   - With interval notation.

   Personally I think it's easist to do the number line before the intervals because it's easy to see the intervals once the number line is shaded.

   Example: $-2x + 8 \geq 18$. Give all three representations of the solution.

   - With inequalities: $x \leq -5$.
   - On a number line:

   - With interval notation: $(-\infty, -5]$

   Example: $5(x - 1) + 3x - 2 < \frac{1}{3}(x + 1)$. Give all three representations of the solution.

3. **Nonlinear Inequalities (One Type):** Aother type of inequality we need to know how to solve is one that is nonlinear. Of course lots of inequalities are nonlinear so this is a bit of an exaggeration. More specifically this type is one where everything can be moved to one side and factored. Sometimes there will be both a numerator and a denominator which can both be factored.

   Example: $\frac{3-x}{x+5} \geq 0$. Take a look at this example. Very often a student will say "Why can't we just multiply both sides by $x + 5$?" and it's a valid question you should address (because it's a typical mistake). The most obvious way for them to understand why not is that we have no idea if $x + 5$ is positive or negative so if we multiplied we'd have no idea whether to flip the direction. Thus we can't.

   Explain to them that the principle we'll be working with is to look at the individual factors and see where they're positive and negative (or zero) and from there see where the whole thing is positive or negative (or zero or undefined). To do this we'll draw a *sign chart*.

Let's draw a sign chart to solve $\frac{3-x}{x+5} \geq 0$:

Instruction: For drawing a sign chart:

(a) Figure out where each factor equals zero. All these x's, when taken in order, break the number line into intervals. List these intervals and the x's at the top of the chart. In our example we have $x = -5$ and $x = 3$.

(b) List the factors down the left and put the entire original espression at the bottom. Thus so far we have:

	$(-\infty,-5)$	-5	$(-5,3)$	3	$(3,\infty)$
$3-x$					
$x+5$					
$\frac{3-x}{x+5}$					

Now then, in each of the entries in the main middle of the chart we write + if the values above are positive on the interval to the left, $-$ if they're negative, and 0 if they're zero. For example in the upper-left blank, the interval is $(-\infty,-5)$. Choose any test point in here, say $x = -7$. Put it into $3-x$ and we get something positive. Thus we put a + in here.

Doing this all over we get:

	$(-\infty,-5)$	-5	$(-5,3)$	3	$(3,\infty)$
$3-x$	+	0	-	-	-
$x+5$	-	-	-	0	+
$\frac{3-x}{x+5}$					

Lastly we fill in the bottom row by looking at the rows above it. For example in the first column we think "positive divided by negative is negative" and put a $-$ in there. We need to be careful here because for example in the fourth column we have "negative divided by zero is undefined" and so for undefined we put in X. Thus we get:

	$(-\infty,-5)$	-5	$(-5,3)$	3	$(3,\infty)$
$3-x$	+	0	-	-	-
$x+5$	-	-	-	0	+
$\frac{3-x}{x+5}$	-	0	+	X	-

(c) So now we want to know where $\frac{3-x}{x+5} \geq 0$. So where is this true? It's true at $x = -5$, on $(-5,3)$ and that's it. So the solution is $[-5,3)$.

Important: Make sure you also sketch this on a number line and also give it with inequalities.

Notes and hints:

- We ask them to use the cut points (like $x = -5$ and $x = 3$) in the chart because otherwise they tend to completely forget them.
- Each row changes either from + to $-$ or $-$ to + only where it $= 0$ so they can cheat on this. It's easy to see where it's $= 0$ and then just check one side.
- The notation we use, $+$, $-$, 0, X is the notation that WebAssign demands which is why we do it this way in class.
- Once they've done a couple of these they actually get very fast at it.

4. Hopefully you'll have time to do another example or at least start setting up the charts. Here are some good ones to start setting up:

Example: $x^2 + 7x + 6 < 0$

Example: $\frac{(x+1)(x-3)}{x(x+7)} \leq 0$

Example: $\frac{6}{x} + \frac{1}{x+1} > \frac{7}{x+2}$

5. **Absolute Value Inequalties:** The final kind (which arises in Calculus and the study of limits) are inequalities involving absolute values.

   Example: $|2x+1| < 5$

   Example: $|1-3x| \geq 10$

   If we always keep the absolute value on the left then the method of solving these breaks down into two types.

   (a) Those with $<$ or $\leq$. Imagine you knew that $|M| < 5$. What could M be? The students will usually see (with some prodding of values of M) that M can between -5 and 5. Thus $|M| < 5$ becomes $-5 \leq M \leq 5$. This is an example of a *compound* inequality and can be solved by dealing with all three portions simultaneously. This would also work if $<$ were replaced by ≤ 5. Here is an example where M is replaced by something else:

   Example: $|2x+1| < 5$

   Example: $|1-x| \leq 10$

   (b) Those with $>$ or $\geq$. In this case work with $|M| > 5$. Likewise it's easy to see that $M > 5$ or $M < -5$ (again, if they don't see this, play a game where you offer them values of M and ask if they satisfy $|M| > 5$, eventually they'll see which do and which don't). Make sure the students see why we can't write this as a single grouping. In this case we work with the two pieces independently and then put the solutions together at the end.

   Example: $|2x+1| > 5$

   Example: $|7-2x| \geq 8$