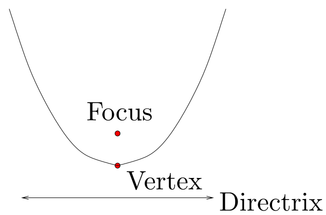
MATH 115 Section 10.1 and 10.2 Lecture Notes

1. **Introduction:** It's worth noting that I teach this differently than the book does, especially regarding ellipses, because the book's exposition is far more confusing, I think.

2. **Parabolas:** The actual definition of a parabola is this: Start with a point called the *focus* and a line called the *directrix*. Take all the points (x, y) which are equidistant from the point and the line.



Note that the parabola wraps around the focus and away from the directrix. The *vertex* is the point directly between them.

We'll only focus (haha!) on parabolas which open up/down and left/right.

- Up/Down: Given focus $(0, p)$ and directrix $y = -p$, the parabola will have equation $4py = x^2$. Note that this is the same as $y = \frac{1}{4p}x^2$, which may be more familiar.

Example: The parabola $8y = x^2$ has $8 = 4p$ so $p = 2$ so the focus is $(0, 2)$ and the directrix is $y = -2$.

- Left/Right: Given focus $(p, 0)$ and directrix $x = -p$, the parabola will have equation $4px = y^2$. Note that this is the same as $x = \frac{1}{4p}y^2$.

Example: What's the equation of the parabola with directrix $x = 5$ and focus $(-5, 0)$?

- With a shift. If the vertex is shifted from the origin the equations become $4p(y - k) = (x - h)^2$ and $4p(x - h) = (y - k)^2$ respectively. This is a shift right by h and up by k. Note if h or k is negative it's left or down.

Example: Consider $f(x) = 2x^2 + 4x$. If we complete the square and do some rewriting:

$$\begin{aligned} y &= 2(x+1)^2 - 2 \\ y + 2 &= 2(x+1)^2 \\ \frac{1}{2}(y+2) &= (x+1)^2 \end{aligned}$$

Therefore this is a shift of $\frac{1}{2}y = x^2$. The shift is down 2 and left 1.
We have $4p = \frac{1}{2}$ so $p = \frac{1}{8}$. Pre-shift the focus is $\left(0, \frac{1}{8}\right)$ and the directrix is $y = -\frac{1}{8}$ and so post shift we have focus $\left(-1, -\frac{15}{8}\right)$ and directrix $y = -\frac{17}{8}$.

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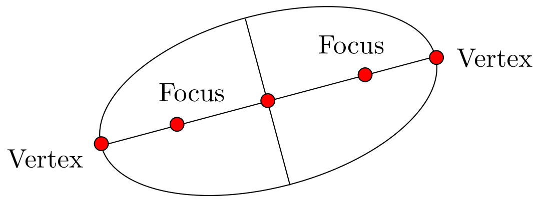
3. **Ellipses:** The actual definition of an ellipse is this: Start with two *foci* and some fixed measurement. Take all the points which have their sum of distances to the foci equal to that fixed measurement.



We'll only focus on ellipses which are centered at the origin.

All ellipses have equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

or can be written this way. The best way to graph these and find the information is this way:

(a) Find the x and y-intercepts by setting $y = 0$ and $x = 0$ respectively.

(b) The longer axis is the *major axis* and the shorter axis is the *minor axis*.

(c) The endpoints of the major axis are the *vertices*.

(d) The foci lie on the major axis. They lie at $\pm\sqrt{|a^2 - b^2|}$.

Example: Find all the info for $49x^2 + 16y^2 = 3136$.

Example: An ellipse has major axis of length 30, foci on the x-axis, and passes through $(7.5, 6)$. Find the equation.

Solution: Note that the first two facts tell us that the x-intercepts are $(\pm 15, 0)$ and hence we have

$$\frac{x^2}{15^2} + \frac{y^2}{b^2} = 1.$$

Then plug in the point to get b.
