MATH 115 Section 2.1-2
Function and Graphs

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1. **Definition:** There are many ways to define a functin but a good intuitive one is this: A function is a collection of inputs, called the *domain* as well as a *rule* which takes each input and produces an output. The collection of all the outputs is called the *range*.

   Example: The domain is all positive numbers and the rule is "multiply by 3 then add 2."

   This is a particularly unwieldy way to write this function so what we generally do is give it a name like "f" and say that f takes any positive number x and it does $3x + 2$ on it. We then write this as $f(x) = 3x + 2$ for $x > 0$. Then we can see for example that $f(1) = 3(1) + 2 = 5$, $f(100) = 3(100) + 2 = 302$ and $f(-3)$ is not defined.

2. Another way of thinking of a function is as a really stupid machine. The function $f(x) = 3x+2$, for example, takes whatever you put inside and tries to do $3x + 2$ on it. But what you put inside need not be a number.

   Examples: $f(t)$, $f(t + 1)$, $f(3x)$, $f(x + h)$, $f(\heartsuit)$ and so on.

3. **Domains:** Notice in the above example that the domain was given: I explicitly stated that $x > 0$ and f(anything else) is not defined. Usually though the domain is not explicitly given and we have to figure it out. This can in fact be very tricky and so we will tend to focus on two particular issues:

   (a) If the function contains a square root then the thing to keep in mind is that the inside (the *radicand*) must be ≥ 0, so take whatever is inside and set it ≥ 0. Solve and the solution is your domain.
   Example: $f(x) = \sqrt{10 - 3x}$
   Example: $g(x) = \sqrt{(x + 3)(x - 5)}$. Don't do it but point out that the solution requires a sign chart.

   (b) If the function contains a fraction then the thing to keep in mind is that the denominator cannot equal 0. The best way to analyze this is to set the denominator $= 0$, solve, and then throw away the results.
   Example: $f(x) = \frac{3}{x-5}$
   Example: $f(x) = \frac{x+1}{x^2-2x-3}$

   (c) If the function contains both issues then we must do both.
   Example: $f(x) = \frac{\sqrt{x+10}}{x-1}$

4. **Graphs:** One of the most useful ways of representing a function is visually. We do this using the graph of a function. Formally, the graph of a function is just a collection of points. The *graph* is the set of points (a, b) with $f(a) = b$.

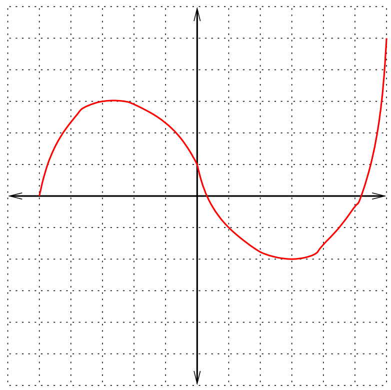
   Example: Show how to graph $f(x) = |x| - x$ by writing a table of points.

5. There are several graphs you should know on sight. These are the following. Draw a quick sketch of each.

   - $f(x) = c$ where c is a constant.
   - $f(x) = mx + b$
   - $f(x) = x^2$
   - $f(x) = x^3$
   - $f(x) = |x|$
   - $f(x) = \sqrt{x}$
   - $f(x) = \frac{1}{x}$

6. Very often when we're discussing functions we simply draw a graph and say that the function is the graph. This is sloppy but common. One of the things we can do with the graph is easily find the domain and range, and find values of the function.

   Example: Given the function $f(x)$ (not explicitly given) with this graph:

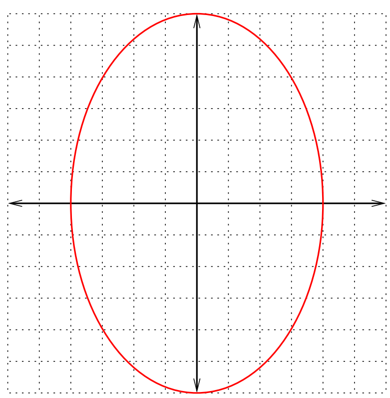
   

   Find $f(0)$, $f(-3)$ and $f(1)$.

   Find all x (approximately) with $f(x) = 1$.

   Find the domain and range of $f(x)$.

7. Lastly, it's important to understand that not every graph is a function. The key to understanding which ones are is to remember that with a function we can only have one y-value for any x-value. A graph like this one:

   has a problem. Consider that $(3, 4)$ and $(3, -4)$ are both on the graph. If this were a function $f(x)$ we'd have $f(3) = 4$ and $f(3) = -4$, but this is not permitted. Thus this is not a function.

   Another way to see the problem is that we can draw a vertical line (in this case $x = 3$) which strikes the graph more than once. If we can do this, the graph is not a function. This is called *the vertical line test*.

   It is worth noting however that this graph still has a domain and range. What are they in this case?