MATH 115 Section 2.3
Increasing and Decreasing Functions, Average Rate of Change

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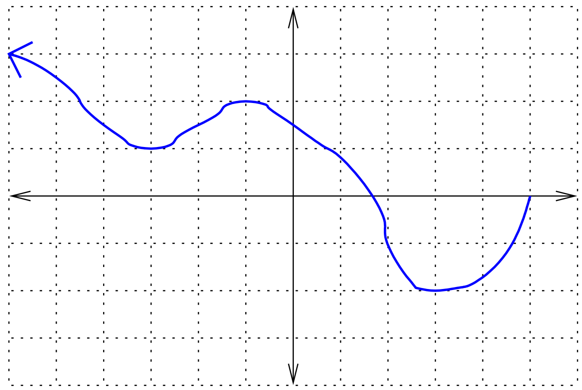
1. **Increasing and Decreasing Functions:** Now that we know what a function is and what a graph is, we'd like to be able to describe them. Look at this graph.



   If you were describing this to your friend over the phone, what might you say? You might say something like:

   It goes down from $x = -\infty$ to $x = -3$, then up from $x = -3$ to $x = -1$, then down from $x = -1$ to $x = 3$, then up from $x = 3$ to $x = 5$.

   But rather than saying "going up" we'll say *increasing* and rather than saying "going down" we'll say *decreasing*. Also rather than saying "from $x = a$ to $x = b$ we'll say "on the interval $[a, b]$ - except when one of them is $\pm\infty$ in which case we use a parenthesis.

   Example: In the above graph, $f(x)$ is increasing on $[-3, -1] \cup [3, 5]$ and decreasing on $(-\infty, -3] \cup [-1, 3]$.

2. **Average Rate of Change of a Function:** Suppose I tell you that $d(t) = t^2 - t + 3$ is the distance I've travelled after t hours. Does this tell you speed? No, it doesn't. Can it tell you anything about speed? For example, $d(0) = 3$ and $d(10) = 93$, so can I conclude anything? Yes, between $t = 0$ and $t = 10$ I travelled 90 miles, so I averaged 9 miles per hour. What we've found here is the *average value of the function*.

   Given a function $f(x)$, the average value of $f(x)$ between $x = a$ and $x = b$ is $\frac{f(b)-f(a)}{b-a}$.

   Example: Find the average value of the function $f(x) = \frac{1}{x}$ between $x = 1$ and $x = 3$.

   Example: Find the average value of the function $g(x) = \frac{x+3}{2}$ between $x = 5$ and $x = 5 + h$. Simplify your answer.