MATH 115 Section 2.4
Transformations of Functions

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1. Now that we know what some basic functions are, we'd like to look at how we can take these basic functions and change them in simple ways, and how we can recognize these changed functions when we see them.

2. **Vertical Shifting:** Consider the function $g(x) = |x| + 3$. What does this look like? If we sketch it by plotting points we see that it looks like $f(x) = |x|$ except it's shifted up by 3 units. This is a vertical shift. We could say that $f(x) + 3$ shifts $f(x)$ up by 3 units.

   > In general if $f(x)$ is a function then $f(x) \pm c$ shifts the function either up if it's $+$ or down if it's $-$ by c units.

   Example: Sketch the graph of $g(x) = \sqrt{x} - 1$.

3. **Horizontal Shifting:** So what if we want to go left or right? Consider $g(x) = (x+2)^2$. If we sketch this we see that it looks like $f(x) = x^2$ except it's shifted left 2 units. Note that $g(x)$ is obtained by *replacing* the x in $f(x)$ by $x + 2$.

   > In general if $f(x)$ is a function then $f(x \pm c)$ shifts the function either right if it's $-$ or left if it's $+$ by c units.

   Example: Draw a graph of $g(x) = (x - 4)^3 - 2$ and have the students identify it as x^3 shifted right 4 and down 2.

4. **Reflections:** Suppose we wish to take a graph and reflect it in either the x or y axis. A really good example for this is the square root function. Let's sketch $g(x) = -\sqrt{x}$ and $h(x) = \sqrt{-x}$. We see that $g(x)$ is what we'd get if we reflected $f(x) = \sqrt{x}$ in the x-axis and $h(x)$ is what we'd get if we reflected $f(x)$ in the y-axis. Note in the latter case that x has been *replaced* by $-x$.

   > In general if $f(x)$ is a function then $-f(x)$ reflects in the x-axis and $f(-x)$ reflects in the y-axis.

   Example: Sketch the graph of $g(x) = -|x + 1|$

5. **Vertical Stretching and Shrinking:** The last thing we wish to do actually alters the shape of the graph, rather than just moving and flipping it. Consider the graphs of $g(x) = \frac{1}{2}x^2$ and $h(x) = 2x^2$. If $f(x) = x^2$ then we see that $g(x)$ is a shrinking of $f(x)$ by a factor of $\frac{1}{2}$ and $h(x)$ is a stretching of $f(x)$ by a factor of 2.

   > In general if $f(x)$ is a function then $cf(x)$ is a shrink if $0 < c < 1$ and a stretch if $c > 1$. Note that if c is negative it's a shrink/stretch *and* a reflection.

   Example: Sketch the graph of $g(x) = 3|x| + 2$. Note that this is a stretch followed by a vertical shift.

   Example: Sketch the graph of $g(x) = -\frac{1}{2}\sqrt{x}$.