MATH 115 Section 2.5 Quadratic Functions

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1. **Intro:** Now that we know how to apply various transformations to a function we'd like to do this to one of the most useful functions in mathematics, the quadratic function.

2. **Definition:** A *quadratic function* is a function of the form $f(x) = ax^2 + bx + c$ with $a \neq 0$.

   Example: $f(x) = 2x^2 - 8x - 6$.

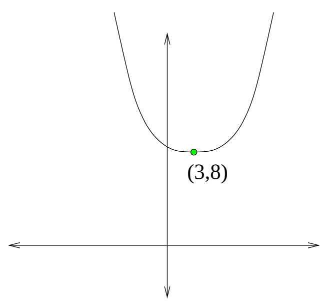
   Note that if we complete the square on our example we find out that $f(x) = 2(x-2)^2 - 14$. Why is this useful? We can see that $f(x)$ is obtained from x^2 by stretching by 2, shifting right 2 and then shifting down 14. This tells us the vertex is at $(2, -14)$.

3. **Furthermore:** In general we can always complete the square to rewrite $f(x) = a(x-h)^2 + k$. This is called the *standard form* (the original one is sometimes called the *general form*). A quadratic function in standard form has the following:

   - Vertex (h, k).
   - Axis of symmetry $x = h$.
   - Opens up if $a > 0$, in which case when $x = h$ then f has a minimum of k.
   - Opens down if $a < 0$, in which case when $x = h$ then f has a maximum of k.

   Example: $f(x) = -3(x+1)^2 + 7$ has vertex $(-1, 7)$ and opens down. Therefore f has a maximum value of 7 when $x = -1$. Sketch it.

   Example: Consider the function shown below. What could it be?

   We know $(h, k) = (3, 8)$ but we don't know a, though we do know $a > 0$. Thus we can only say $f(x) = a(x-3)^2 + 8$ with $a > 0$.

   Example: Sketch the graph of $f(x) = -3(x-2)^2 + 6$. Find and label the vertex, axis of symmetry and all the intercepts.

4. **Lazy:** Suppose then you're given $f(x) = x^2 - x - 2$ and you don't want to complete the square. Is there a short-cut to finding the vertex? If the only thing you need is the vertex then maybe there's a nice way of picking it out. The answer is "yes".

   A quadratic function in standard form $f(x) = ax^2 + bx + c$ has the following:

   - The x-coordinate of the vertex is $x = -\frac{b}{2a}$.
   - The y-coordinate of the vertex is $y = f\left(-\frac{b}{2a}\right)$.
   - Axis of symmetry $x = -\frac{b}{2a}$.
   - Opens up if $a > 0$, in which case when $x = h$ then f has a minimum of k.
   - Opens down if $a < 0$, in which case when $x = h$ then f has a maximum of k.

   Example: Let $f(x) = 2x^2 + 6x + 10$. Where is the vertex? Does it open up or down? Does it have a minimum or a maximum? What is it and where does it occur? Graph this function, including all intercepts.

5. **Application:** Quadratic functions are very useful. Here is a simple application.

   Example: Suppose a ball is thrown with an initial velocity directly upwards of 96 feet per second and an initial height of 4 feet. Physics tells us that after t seconds its height will be $h(t) = -16t^2 + 96t + 4$ feet. What does the vertex tell us?

   Answer: The vertex is located at $t = -\frac{96}{2(-16)} = 3$ and $h(3) = -16(3)^2 + 96(3) + 4 = 148$. Since $a = -16 < 0$ we know that when $t = 3$ the function $h(t)$ achieves a maximum value of 148. In our application this means that after 3 seconds the ball is highest at 148 feet up.