MATH 115 Section 2.6
Applications with Functions, Maximizing and Minimizing

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1. **Note:** I'm going to briefly discuss the vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ of the quadratic function $f(x) = ax^2 + bx + c$ here and use it to lead into these examples, since I didn't get to that part of section 2.5 yesterday.

2. **Intro:** So now we know how to find the maximum or minimum of a quadratic function, we can see how this may be useful. If we are trying to maximize or minimize some function like height, area, distance and so on, all we need is a function which yields that quantity. To elaborate let's work slowly through an example:

3. **Applications:** A poster has perimeter 14 feet. Suppose the length is x. What's the largest poster we could have?

   Note that we want to maximize *area* so let's find a function for area. Area is length times width so we could write $A = xy$ but this function has two variables and we want one, just x. But note that $2x + 2y = 14$ and so $y = 7 - x$ and hence $A = x(7 - x)$. Now area is a function of just x, so we could write $A(x) = -x^2 + 7x$. Note that in this function length x goes in and area $A(x)$ comes out. Note also this is a quadratic function which opens down and hence has a maximum. The vertex has $x = -\frac{b}{2a} = \frac{7}{2}$ (don't worry about $A(x)$ right now.) So let's think about this. If $x = \frac{7}{2}$ the $A(x)$, the area, is maximum. The other dimension is then $y = 7 - \frac{7}{2} = \frac{7}{2}$.

   Note that the key steps here are:

   - Write down a formula for the thing you want to maximize/minimize.
   - Get it in terms of a single variable. Sometimes this is given to you, sometimes it's not.
   - Rewrite as $f(x) = ax^2 + bx + c$.
   - Find and interpret the vertex.

4. **More Examples:**

   Example: A farmer wants to fence in a field with a divider across the width. The fencing around the outside costs \$15/yard and the fencing down the middle costs \$10/yard and unfortunately the farmer has only \$3000 to spend. First find a function which gives the area as a function of the width x. Then, what should the dimensions be in order to maximize the area?

   Example: Find two numbers whose sum is 35 and the sum of whose squares is minimum.

5. **Lastly:** It's worth noting that in theory the function need not be quadratic, though if this is the case we have no method of finding the minimum.

   Example: A box is built such that the width is half the length and the height and twice the length add to 100in. Find a function which models the volume as a function of x, the length.

   Solution: Since $V = xyz$ with $x =$length, $y =$width and $z =$height, we have $y = \frac{1}{2}x$ and $z + 2x = 100$ so $z = 100 - 2x$. Therefore $V(x) = x\left(\frac{1}{2}x\right)(100 - 2x) = -x^3 + 50x^2$ but since this is not a quadratic we can't find out how to make the volume either maximum or minimum or if it's even possible.