MATH 115 Sections 3.1a Lecture Notes
Polynomials - Part 1 of 2

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1. **Introduction:** Precalculus, as a whole, is basically the study of equations and functions and what you can do with them. Since we've learned what the graph of a function is we'd now like to look at the graphs of specific types of functions. Sections 3.1 and 3.6 begin this process; section 3.1 looks at polynomial functions and section 3.6 looks at rational functions.

2. **Basic Definitions:**

   (a) A *monomial* is a constant or a constant multiplied by a whole number power of x.
   Examples: 3, $2x$, $5x^7$, $-\frac{1}{2}x^{10}$, $\frac{x^3}{7}$

   (b) A *polynomial* is a bunch of monomials added or subtracted together.
   Examples:

   (c) The *degree* of a polynomial is the highest power of x.
   Examples:

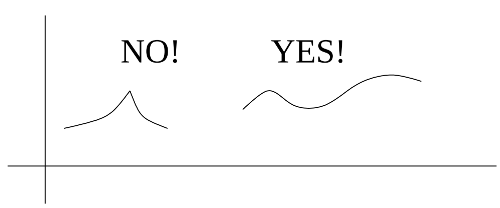
   (d) The *leading term* of a polynomial is the monomial part which includes the highest power of x. It includes the $\pm$ in front.
   Examples:

   (e) The *leading coefficient* of a polynomial is the constant part of that monomial.
   Examples:

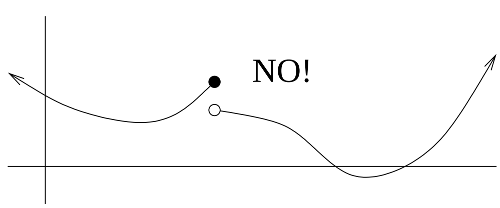
   (f) The *constant term* or *constant coefficient* is the monomial part with no power of x. If there isn't one, it's 0.
   Examples.

3. **General Description of Graphs:** We will explain some of these facts but some must wait for calculus. There are several features of polynomial graphs which we must always keep an eye on:

   (a) They are *smooth*, meaning no sharp corners.

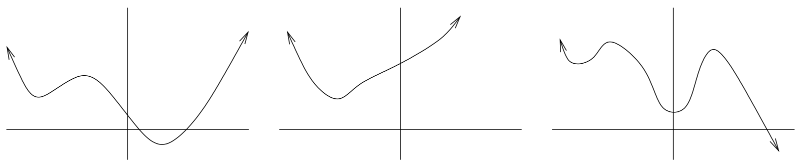


   (b) They are *continuous*, meaning no breaks.



   (c) To the left and to the right the graph goes on forever - it does not stop.

(d) A *turning point* is a place where the graph turns around. If the degree is some number n then the graph can turn around at most $n-1$ times. Example: The graph of $f(x) = 5x^4 - 3x^2 + 1$ has degree 4 and therefore has at moat $4 - 1 = 3$ turning points. Thus it might look like (a) or (b) but could not look like (c).



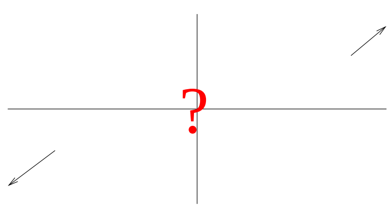
(a) (b) (c)

4. **Intro to End Behavior:** We study the graphs of polynomials in two stages: End behavior and middle behavior. First we look at what happens to the far left and right of the graph. In other words, as x gets really big, what happens? As x gets really far negative, what happens? We'll have a rule but for now look at this example:

   Example: Let $f(x) = 2x^3 + x + 1$. Let's list some values of x and for $f(x)$.

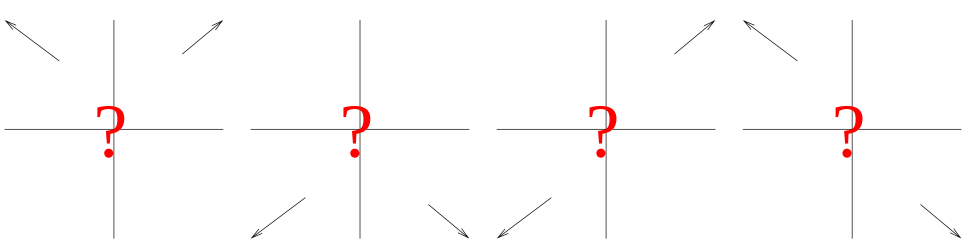
x	$f(x)$
-100	$-2000000 - 100 + 1$
-10	$-200 - 10 + 1$
10	$200 + 10 + 1$
100	$2000000 + 100 + 1$

   Thus to the right (as x gets more positive) we see that $f(x)$ does too and so it goes up and up. To the left it goes down and down. Even though we don't know what it does in the middle we can say that the ends look something like this:



   The point is (and we see this from our list) that the leading term is the all important term because as x goes more positive or more negative the other terms pale in comparison. Thus the end behavior is determined by the degree of the leading term and the coefficient of the leading term. There are four possibilities:

5. **Leading Term Test:** Examine two things: Is the leading term positive or negative? Is the degree even or odd? Then:



LT + Deg even | LT − Deg even | LT + Deg odd | LT − Deg odd

   Examples: