1. **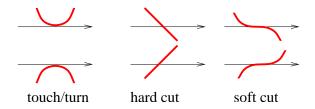First a Shortcut:** In testing end behavior we need to know the degree and the leading coefficient. This is easy to see if the polynomial is like $p(x) = -4x^7 + 3x + 3$, but what if the polynomial is written like $p(x) = (x + 1)(-3x - 1)^2(5x - 1)$? Note that this is indeed a polynomial because if you muliplied it all out it would fit the description. So how can we get the degree of this thing without muliplying it out?

    The shortcut is that if we look at each factor and we drop out all except the leading term of each factor and if we multiply the rest together we will get the leading term of the original polynomial.

    Example: If $p(x) = (x + 1)(-3x - 1)^2(5x - 1)$ we drop out the $+1$, -1 and -1 to get $(x)(-3x)^2(5x)$ which is $45x^4$, so the leading term of $p(x)$ is $45x^4$ so it goes up at both ends.

2. **Middle Behavior:** So far we know how to analyze the end behavior of a polynomial so now we need to look at the middle. There are two critical things, x and y-intercepts. The y-intercept is easy, simply put $x = 0$ and find $p(0)$.

3. **x-intercepts:** Consider the function $p(x) = (2x + 4)(3x - 12)^3$. Finding the x-intercepts is easy, simply set the whole thing equal to 0 and solve, or really just set each factor equal to 0 and solve. The x-intercepts are clearly $x = -2$ and $x = 4$. But what do these look like? Note that an x-intercept can look like three different things: (Each vertical pair is considered "the same".)

    touch/turn   hard cut   soft cut

    So the question is, at each intercept, what happens?

    Definition: Observe that the intercept $x = -2$ came from the factor $(2x + 4)$ and this is to the power 1. This is called the *multiplicity* of the intercept. Similarly the intercept $x = 4$ has multiplicity 3.

    The multiplicity dictates the behavior. The rule is:

    - If $m = 1$ then it is a hard cut.
    - If $m =$ any other odd number then it is a soft cut.
    - If $m =$ even then it touches and turns.

    Since $x = -2$ has $m = 1$ it's a hard cut and since $x = 4$ has $m = 3$ it's a soft cut. This coupled with the end behavior (the leading term is $(2x)(3x)^3 = 54x^4$ so it's up at both ends) and the y-intercept $(p(0) = (4)(-12)^3 = 3456)$ allows us to draw the graph.

4. **Examples:** Here are some more examples. Find the end behavior (from the leading term), the y-intercept (plug in $x = 0$) and the x-intercepts and behavior and use these to draw the graph.

    Example: $p(x) = -3(x + 1)^2(x - 5)$ (Don't forget the -3 when looking at the leading term!)

    Example: $p(x) = x(2x + 7)^5(8 - x)^4$

    Example: $p(x) = x^4 - 7x^3 + 6x^2$