MATH 115 Sections 4.2 Lecture Notes
Logarithmic Functions

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1. **Introduction:** We'd like to do for logarithmic functions what we did for exponential functions, learn the basic graphs and transformations. First though we need to know exactly what the logarithmic function actually is.

2. **Definition:** Define the *logarithm function with base b* denoted $\log_a x$ to be the power that a is raised to in order to obtain x.

   Example: $\log_2 8 = 3$ because $2^{\boxed{3}} = 8$.

   Note: Essentially these are the same equation written in two different ways. We say $\log_2 8 = 3$ is in *logarithmic form* while $2^3 = 8$ is in *exponential form*.

   Example: $\log_{25} 5 = \frac{1}{2}$ because $25^{1/2} = 5$.

   Example: $\log_{1/3} 9 = -2$ because $\left(\frac{1}{3}\right)^{-2} = (3)^2 = 9$.

   Example: $\log_2 5 =$ nothing nice ($2^? = 5$) but since $2^2 = 4$ and $2^3 = 9$, probably $\log_2 5$ is between 2 and 3.

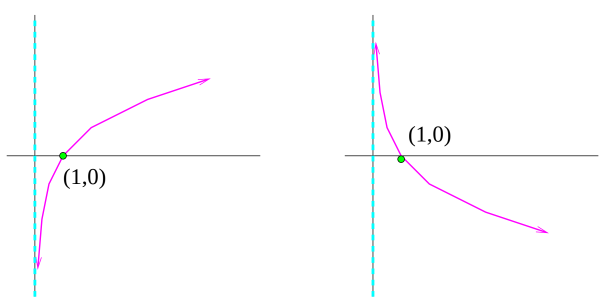
   Another way to see this that: $\log_a x = y$ iff $a^y = x$.

   So in other words, the logarithmic function is the inverse of the exponential function. That is, $\log_2 x$ is the inverse of 2^x and $\log_3 x$ is the inverse of 3^x and so on.

3. **Special Logarithms:** Because they are so frequently used, $\log_{10} x$ and $\log_e x$ get special notation. In place of $\log_{10} x$ we write $\log x$ (this is called the *common logarithm*) and instead of $\log_e x$ we write $\ln x$ (this is called the *natural logarithm*).

   Example: $\log 100 = 2$ because $10^2 = 100$.

4. **Graphs:** Just as for exponential function there are two categories of graphs, those when $a > 1$ and those when $0 < a < 1$. These two versions look like:

   The one on the left is $f(x) = \log_a x$ for $a > 1$ (including $\log x$ and $\ln x$) and the one on the right is $f(x) = \log_a x$ for $0 < a < 1$.

   Note the important features:

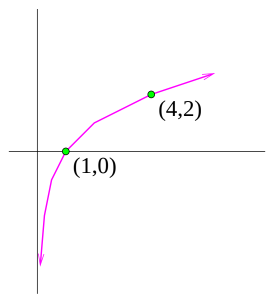
   - The left one is increasing, the right is decreasing.
   - Passes through $(1, 0)$ (because $f(0) = \log_a 1 = 0$ because $a^0 = 1$ for any a.)
   - Vertical asymptote at $x = 0$.
   - Domain $(0, \infty)$ and range $(-\infty, \infty)$.

5. **Graphing:** Transformations of the basic logarithmic graphs work like all other transformations. Here are some examples:

   Example: Sketch $g(x) = 3 + \log_2 x$. This is $\log_2 x$ shifted up 3. It's worth noting that we often write the $3+$ in front because if we write $\log_2 x + 3$ it's not entirely clear if the $+3$ is inside the logarithm. Once you've graphed it, note all the relevent details.

   Example: Sketch $g(x) = \ln(-x + 1)$. This is $\ln x$ shifted left 1 then reflected in the y-axis. Note all the relevent details.

   Example: Find the function of the form $f(x) = \log_a x$ which looks like:



   Final Note, Domain: The domain of $\log_a x$ from the picture is $x > 0$. In general the stuff inside the logarithm (the argument) must be greater than 0.

   Example: The domain of $f(x) = \log_2(x - 3)$ is $x - 3 > 0$, or $x > 3$.

   Example: The domain of $f(x) = \log_2(x^2 - x - 6)$ is $x^2 - x - 6 > 0$ which requires a sign chart to solve.