MATH 115 Sections 4.4 Lecture Notes
Exponential and Logarithmic Equation

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1. **Introduction:** Consider these two problems:
   - Suppose $P(t) = 100e^{0.012t}$ gives the population of a city (in thousands) after t years. When will the population be $200,000$? To answer this we have to solve $100e^{0.012t} = 200$. This is an example of an exponential equation.
   - Suppose we wanted the y-intercept of $f(x) = 2 - \log_3(x+1)$. To answer this we have to solve $2 - \log_3(x+1) = 0$. This is an example of a logarithmic equation.
2. **Definition:** An *exponential equation* is an equation in which the desired variable is inside an exponent.

   The method of solution is very direct:
   (a) Isolate the exponential expression, meaning get everything to one side except the base and exponent. In the above example we'd divide by 100 to get $e^{0.012t} = 2$.
   (b) Convert from an exponential equation to the equivalent logarithmic equation. In the above example this becomes $0.012t = \ln 2$.
   (c) Solve. In the above example we get $t = \frac{\ln 2}{0.012}$.

Example: Solve the exponential equation $5 \cdot 3^{2x-1} = 20$.

Example: The velocity of a skydiver t seconds after jumping is given by $v(t) = 80\left(1 - e^{-0.2t}\right)$. After how many seconds is the velocity 70 ft/sec.?

Example: Find the x-intercept of $f(x) = 2^{x-3} - 5$. Draw a quick sketch of the graph and mark this point.

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3. **Definition:** Similarly, a *logarithmic equation* is an equation in which the desired variable is inside one or more logarithms.

   The method of solution is almost as direct as for exponential equations with only simple additional first and last steps:
   (a) If there are several logarithms first combine them with the rules of logarithms.
   (b) Isolate the logarithmic expression.
   (c) Convert from a logarithmic equation to an exponential equation.
   (d) Solve.
   (e) Sometimes extraneous solutions appear. These are solutions which arise during the solving process but don't really work. Check your solutions to see if they're in the domains of all the logarithms in the original equations. If not, throw them out.

Example: Solve the equation $\log_5(x+1) - \log_5(x-1) = 2$.

Example: Solve the equation $\log(x+9) + \log(x+6) = 1$.

Example: Find the y-intercept of $f(x) = 2 - \log_3(x+1)$. Draw a quick sketch of the graph and mark this point.
